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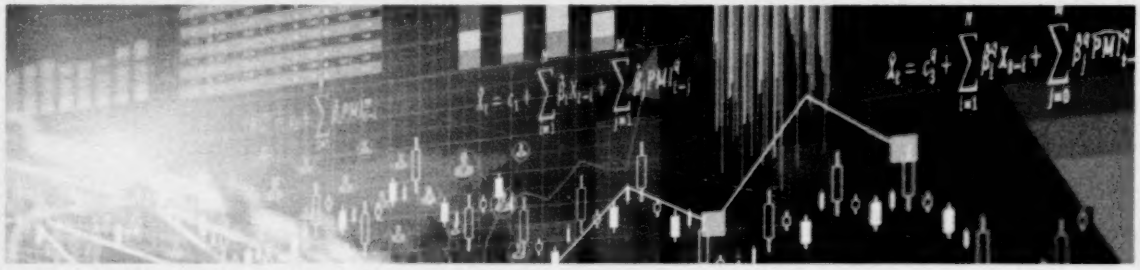
Monetary Policy Tradeoffs Between Financial Stability and Price Stability



by Malik Shukayev and Alexander Ueberfeldt

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Monetary Policy Tradeoffs Between Financial Stability and Price Stability

by

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Abstract

We analyze the impact of interest rate policy on financial stability in an environment where banks can experience runs on their short-term liabilities, forcing them to sell assets at fire-sale prices. Price adjustment frictions and a state-dependent risk of financial crisis create the possibility of a policy tradeoff between price stability and financial stability. Focusing on Taylor rules with monetary policy possibly reacting to banks' short-term liabilities, we find that the optimized policy uses the extra tool to support investment at the expense of higher inflation and output volatility.

Bank topics: Monetary policy framework; Financial stability; Transmission of monetary policy
JEL codes: E44, D62, G01, E32

Résumé

Nous analysons l'incidence d'une politique de taux d'intérêt sur la stabilité financière dans un contexte de panique où les banques peuvent devoir rembourser un volume anormalement élevé de titres de dette, ce qui les obligerait à vendre des actifs à des prix de liquidation. Les frictions créées par le coût d'ajustement des prix et un risque de crise financière qui dépend de la conjoncture pourraient nécessiter un arbitrage entre la stabilité des prix et la stabilité financière. Appliquant une règle de Taylor qui incorpore une possible réaction de la politique monétaire face à l'évolution du passif à court terme des banques, nous constatons que les mesures de politique optimales font appel à l'outil supplémentaire pour soutenir l'investissement mais s'accompagnent d'une inflation plus élevée et d'une production volatile.

Sujets : Cadre de la politique monétaire; Stabilité financière; Transmission de la politique monétaire
Codes JEL : E44, D62, G01, E32

Non-technical summary

Before deposit insurance was introduced, bank runs and ensuing fire sales were frequent and disruptive events for banks and other financial institutions and had sizable negative implications for the economy. After a long quiet period, a new source of fragility emerged when financial institutions changed the composition of their short-term liabilities away from government-insured retail deposits toward uninsured wholesale funding. The 2007–09 financial crisis highlights the risks associated with this new source of funding, as financial institutions and corporations rapidly withdrew their funds at the first sign of balance sheet fragility. During the run on the wholesale market, costs of funding for affected institutions soared. More generally, such banking crises are costly because they are typically associated with large output losses and increases in fiscal debt. This raises the question: to what extent can policy support financial stability and reduce the probability of such crises?

Extending Stein (2012), we build a quantitative model calibrated to the United States. As in Stein's paper, banks issue short-term liabilities that are valued by households for being highly liquid. The liquid nature of these liabilities is enforced by a borrowing constraint on the maximum amount of banks' short-term liabilities. When funds are withdrawn prematurely, banks are forced to sell claims to their assets at a fire-sale price. Because banks do not internalize their influence on that price, a fire-sale externality arises, leading to excessive short-term debt levels relative to the social optimum. This excessive short-term borrowing leaves the financial system vulnerable to costly financial crises. Our main extensions of Stein (2012) can be summarized as follows. First, we make the probability of bank runs state-contingent by linking it to the probability of insolvency in a way that is consistent with the equilibrium. Second, we embed the model in an infinite horizon dynamic stochastic general equilibrium framework with nominal price rigidities, thus forcing monetary policy to consider both price stability and financial stability at the same time. Monetary policy is set by a standard Taylor rule that is augmented to respond to short-term funding.

Besides an aggressive response to inflation fluctuations and a weak response to economic activity, the optimized augmented Taylor rule features some reaction to fluctuations in banks' short-term liabilities. This rule manages the tail risk by containing short-term funding but accepts both increases in financial vulnerability and sharp decreases in inflation during financial panics. Intuitively, the key task for monetary policy is to support the flow of savings into investments; disruptions to this channel are much more harmful for long-term welfare than transitory increases in the probability of a crisis and in the volatility of inflation and output. This finding emphasizes the greater importance of a forceful policy response to fire sales during a crisis relative to managing the probability of a crisis or the pricing friction.

Interestingly, a more standard optimized policy rule, with no direct response to financial variables, attains lower volatilities of inflation and output, marginally reducing the average probability of a financial crisis at the same time. This gain in economic stability, however, comes at the expense of lower wealth, consumption and welfare since short-term funding remains relatively high, making crises and fire sales more costly.

1 Introduction

This paper analyzes the tradeoff between price stability and financial stability a central bank might face when dealing with financial sector misalignments and concerns about inflation. To speak meaningfully about this issue, we develop a model with standard pricing frictions and a pecuniary externality, where the latter arises from the possibility of bank runs that trigger fire sales of assets by banks. Such banking crises arrive with an endogenous probability and have severe macroeconomic implications for output and inflation. Both the probability of a crisis and the resulting output losses are increasing in the share of run-prone liabilities on banks' balance sheets. Hence, an interest rate policy can reduce the risks and the costs of banking crises by responding to such liabilities.

Interestingly, we find that the option to respond to commercial banks' run-prone funding will not lead to higher inflation and output stability. Instead central banks will aim to stabilize investment. This insight is consistent with Issing's (2003) notion of financial stability, defined as "the prevalence of a financial system, which is able to ensure in a lasting way, and without major disruptions, an efficient allocation of savings to investment opportunities." As anticipated by Issing, financial stability in this sense comes temporarily at the expense of sizable deviations of inflation and output from their long-term trends.

The 2007–09 financial crisis has shown that bank runs and associated fire sales of assets are still a relevant concern for policy-makers. While deposit insurance seemingly eliminated bank panics caused by household deposit withdrawals, new types of short-term liabilities, prevalent among financial institutions, have become a source of financial fragility. Figure 1 shows that financial crises associated with bank runs were quite common in the United States before deposit insurance was introduced. The red spikes in Figure 1 mark the occurrences of financial crises, and the green line shows when deposit insurance was introduced. These crisis episodes were often accompanied by economic downturns, as can be seen from the labour productivity series in Figure 1.

After a long quiet period without crisis, the change in the composition of short-term funding, from retail to wholesale, shown in Figure 2, has created a very reactive type of financial liability, as was evident during the 2007–09 financial crisis. Gorton and Metrick (2012) highlights the risks associated with wholesale funding created by short-term fire sales in the context of the recent financial crisis. As the signs of financial sector distress mounted, financial institutions and corporations started to run on distressed intermediaries, rapidly withdrawing their funds or increasing their collateral requirements. Figure 3 illustrates how the costs of repo refinancing grew from almost zero in July 2007 to 46 per cent haircuts by January 2009.[1]

Building on the model of bank runs and fire sales of assets proposed in Stein (2012), we analyze the nature of the monetary policy tradeoff in a run-prone environment.[2] Our model features banks

[1] For a more detailed discussion of the relationship between bank funding and financial crises, see Gorton (2012).

[2] Monetary policy might not be the ideal tool to deal with such problems, however, as Stein (2013) highlighted, because of regulatory arbitrage, gaps in the regulatory framework or the speed with which regulation can react, monetary policy might still have an important role in this context. Also important is the fact that monetary policy

that issue short-term nominal liabilities, which are valued by households for being riskless and redeemable on demand. When these funds are withdrawn prematurely, banks are forced to sell claims to their assets at a fire-sale price. The safe nature of these liabilities is enforced by a borrowing constraint on the maximum amount of banks' short-term liabilities. This borrowing constraint, combined with a standard fire-sale externality, creates the possibility of financial sector misalignments in our model. Specifically, banks issue too much short-term debt relative to the socially optimal level, leaving the financial system vulnerable to costly financial crises.

In our model, as in Stein (2012) and Gertler and Kiyotaki (2013), the output costs of financial panics depend on the amount of short-term liabilities on the balance sheets of financial institutions. The more short-term funding banks attract, the bigger the disruptions to production created by bank runs and fire sales of assets.

Our extensions of Stein (2012) can be summarized as follows. First, we make the probability of bank runs state-contingent by linking it to the probability of insolvency in an equilibrium-consistent way. The state-contingent bank run probability in our model gives monetary policy the best chance to matter for financial stability because it is in a position to make crises unlikely tail events. Second, we embed the model in an infinite horizon framework with standard nominal price rigidities, thus forcing monetary policy to consider concerns about both price and financial stability. Overall, this addition allows us to better understand the business cycle properties of financial risk and to analyze the potential monetary policy tradeoffs. Third, we allow for risk-aversion on the part of households. Fourth, we allow for markup shocks and liquidity demand shocks, which help us to replicate data moments for inflation and interest rate spreads.

The model is calibrated to the US data from 1986Q1 to just before the 2007–09 financial crisis, using macroeconomic time series and benchmarking against cross-sectional data from the balance sheets of publicly listed US banks.

Focusing on Taylor style monetary policy rules, the main findings of our paper are the following: the optimized monetary policy (OMP) rule is aggressive on inflation but is responsive to fluctuations in banks' short-term funding positions at the same time. The optimal coefficient on short-term funding is negative, which means the policy rate is decreasing in short-term funding. This policy response to short-term funding may seem destabilizing at first. Indeed, other things equal, when the policy rate falls relative to long-term rates, the banks have more incentive to fund themselves short-term and thus more cheaply. For households, however, wider interest rate spreads increase the opportunity cost of short-term assets, thus moderating their supply of short-term funding to the banks. In our quantitative experiments, we find supply effects of short-term funding dominate demand effects. Thus a negative coefficient on short-term funding in the central bank's policy rule has a stabilizing effect on the quantity of run-prone liabilities by choking households' supply of short-term funding. More importantly, the tighter control over the short-term funding confers the long-term benefit of an adjustment in private sector expectations. Just like an aggressive response

is able to influence all financial sector decision, or, as Stein phrased it, to get "in all of the cracks" left by regulation.

to inflation improves the tradeoff between inflation and output, we find that an aggressive response to the household supply of short-term funds allows the central bank to improve its tradeoff between the level of interest rates and the amount of run-prone liabilities accumulated by banks. The mere threat of an aggressive response to short-term funding is sufficient to moderate the accumulation of run-prone liabilities. In our simulations with an OMP, the ability of the central bank to control short-term funding lowers the average interest rates relative to the benchmark monetary policy (BMP), or calibrated, economy. The lower borrowing rates stimulate investment and wealth accumulation, leading to a substantial welfare gain. Thus, our main finding is that a tighter control over short-term funding improves the central bank's policy tradeoff between the average level of investment and the amount of financial fragility. It is not a free lunch, however. We find that under the optimal policy, the central bank needs to accept higher levels of inflation and output volatility relative to the BMP economy. Thus, there is still a policy tradeoff between price stability and the financial stability, as defined in Issing (2003).

To asses the importance of the central bank's ability to respond to short-term funding, we also examine the restricted optimized policy rule (ROMP), which precludes a direct response to banks' short-term funding. The ROMP is more cautious than its unrestricted counterpart (OMP). Unable to contain short-term funding more directly, the ROMP focuses primarily on more standard macroeconomic stability goals, such as reducing the volatility of inflation and output. Moreover, the ROMP also lowers the average crisis probability, relative to the OMP. However, these gains in macroeconomic stability come at the expense of lower average output, investment and wealth, especially in the aftermath of crisis episodes. Overall, the restriction implies a substantial welfare loss relative to the unrestricted optimized policy.

To understand the nature of high-risk periods and the way monetary policy tries to manage them, we characterize the average conditions of high-risk periods relative to the conditions in all other periods. We find that high-risk periods exhibit lower than steady state wealth, output, inflation and interest rates. OMP and ROMP reduce the frequency of such tail events and the conditions under which they arise. However, given that the probability of the crises is still quite small, even in these high-risk periods, it appears more promising to take a systematic approach to managing the consequences of financial crises ex post than trying to target them ex ante.

Our paper contributes to the recent literature on the role of "leaning against the wind" in the objective function of monetary policy.[3] Within this line of research, our paper is closest to Angeloni and Faia (2013). The authors provide a tractable model in which the probability of a bank run depends on banks' leverage, which in turn might be suboptimally high from a social perspective because bankers are less risk averse than households.[4] Angeloni and Faia (2013) find that the

[3] See Borio (2014) for a recent account of historical record regarding "leaning" as a monetary policy approach.

[4] A recently emerging literature uses empirical estimates of the link between household debt and the probability of a crisis to analyze the gains from monetary leaning against the probability of a crises (see Svensson [2016], Ajello et al. [2015] and Alpanda and Ueberfeldt [2016]). Abstracting from investment decisions and an active financial sector, these papers generally find small or no gains from leaning.

marginal benefit of leaning against buildups in asset prices is small, provided that monetary policy responds aggressively to fluctuations in inflation. Another difference from our paper is that Angeloni and Faia do not consider fire sales as part of their model, instead they assume a fixed exogenous cost of bank runs. In our model, as in Stein (2012), the cost of bank runs depends on banks' funding choices, which are influenced by monetary policy.

Another closely related paper is Brunnermeier and Sannikov (2014), who provide an analytical characterization of global non-linear dynamics in a tractable continuous time model with financial frictions and fire sales of assets. With respect to some core elements, e.g. large shocks that require global solution methods, resource misallocation resulting from liquidity problems and fire sales, our analysis is similar to theirs. We do not provide an analytical characterization, however, aiming instead for a quantitative model that replicates key characteristics of the US macroeconomic environment.

Other papers that consider leaning against an accumulation of financial liabilities are Ajello et al. (2015) and Alpanda and Ueberfeldt (2016). Both papers approximate the probability of a crisis using reduced-form parametric functions, thus facing concerns regarding the policy invariance of the probability function parameters. Abstracting from problems related to bank funding, they find that the benefits of monetary policy responses to asset price misalignments are small at best. In contrast, our analysis finds substantial welfare benefits from a monetary policy response to the banks' short-term liabilities, i.e. OMP.

The paper is organized as follows: Section 2 describes the model; calibration details are in Section 3; Section 4 presents the policy experiments and other results; Section 5 concludes.

2 A model of bank runs, fire sales and monetary policy

The model has six decision makers: banks, patient investors, intermediate-good producers, retailers, households and the government.

The heart of the model is the financial sector that admits bank runs and fire sales of assets. The financial sector consists of two types of intermediaries: banks and patient investors. Banks make irreversible investments into illiquid productive assets before learning whether there will be a bank run on their demand deposits. Unlike banks, patient investors can wait until the bank-run shocks have realized before investing in illiquid assets. The presence of patient investors is crucial for the existence of fire sales because it allows banks to raise funds when short-term funds are withdrawn prematurely. Specifically, banks can sell part or all of their illiquid investments to patient investors, who still have liquid funds in hands, during a financial crisis.[5]

Firms that produce intermediate goods require external financing and obtain the funds from the financial sector. For computational tractability and to focus attention on the fire-sale externality, we follow Stein (2012) and abstract from potential asymmetric information problems arising from

[5]The government could play a comparable role to that of patient investors in the event of a crisis.

the external funding of firms that produce intermediate goods. We assume that these producers are controlled by banks or patient investors, depending on their source of funding. Thus banks and patient investors collect investment returns from their intermediate-good firms, and use these proceeds to pay deposit returns, bond returns and profits.

Households consume final goods and lend their savings to banks and patient investors. In addition, households' labour services are inelastically supplied to the intermediate-good producers.

To introduce monetary policy, we model monopolistically competitive retail firms, who buy intermediate goods, differentiate them and then sell final goods to the households. When setting prices for their goods, retailers face quadratic costs of price adjustment, as in Rotemberg (1982).

The government controls the nominal short-term interest rate, setting it in accordance with a simple log-linear rule, as in Taylor (1993).

2.1 The household problem

The representative household's problem can be written in a sequential form:

$$V\left(W_t, z_t, \gamma_t, \eta_t\right) = \max_{\substack{C_t, D_t^h, \\ A_t, W_{t+1}}} u\left(C_t\right) + \gamma_t v\left(\frac{D_t^h}{P_t}\right) + \beta \sum_s \Pr\left(s\right) V\left(W_{t+1}, z_{t+1}, \gamma_{t+1}, \eta_{t+1}\right)$$

subject to

$$P_t C_t + \frac{D_t^h}{R_t} + \frac{A_t}{R_t^A} \leq P_t W_t \tag{1}$$

$$P_{t+1} W_{t+1} \leq D_t^h + A_t + \Pi_{t+1} + T_{t+1} + P_{t+1}\left(w_{t+1}^P + w_{t+1}^B\right) \tag{2}$$

$$\log\left(\gamma_t\right) = \log\left(\bar{\gamma}\right) + \rho_\gamma \log\left(\gamma_{t-1}\right) + \varepsilon_{\gamma,t}$$

$$\log\left(\eta_t\right) = \log\left(\bar{\eta}\right) + \rho_\eta \log\left(\eta_{t-1}\right) + \varepsilon_{\eta,t} \tag{3}$$

$$\log\left(z_t\right) = \log\left(z_{t-1}\right) + \varepsilon_{z,t} \tag{4}$$

where W_t is the real value of financial wealth at the beginning of period t, D_t^h are the household demand deposits held by banks, while $A_t = A_t^P + A_t^B$ are bonds issued by banks or patient investors.

Households supply two units of labour services, divided equally between intermediate-good producers funded by banks or patient investors, and receive real wages w_{t+1}^B and w_{t+1}^P in return.[6] Π_{t+1} are nominal dividends from banks, patient investors, and retailers, and T_{t+1} are lump-sum nominal transfers from the government.[7]

R_t is the state-uncontingent nominal interest rate on demand deposits and R_t^A is the state-uncontingent nominal interest rate on bonds. Bonds are assumed to be riskless from a household

[6] For simplicity, we assume segmented labour market, with the household sector supplying one unit of labour to each intermediate-goods producer, which is funded by a bank or a patient investor.

[7] These transfers contain bankruptcy proceedings, should the banks still hold assets after a run, as well as seigniorage.

perspective because either bank equity holders or the government will pay in the event that the business is unable to.[8] This assumption simplifies our analysis, but is not essential for the results.[9] The term $\gamma_t v\left(D_t^h/P_t\right)$ represents the utility value (or liquidity service) of a risk-free asset ($v'\left(D_t^h/P_t\right) > 0$ and $v''\left(D_t^h/P_t\right) < 0$). The variable γ_t is a stochastically varying liquidity preference parameter. The other relevant shocks in the household problem are the aggregate productivity, z_t, and the markup shock, η_t.

Finally, Y_t is a aggregate good and P_t is the aggregate price level. The households buy a continuum of goods, $\int_0^1 P_t(i) Y_t(i)\, di$ from retailers and combine them into a final good according to a constant elasticity of substitution aggregator to

$$Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{1}{\varepsilon-1}}.$$

A household's optimal demand for individual good $Y_t(i)$ depends on its relative price

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t,$$

where

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

is the welfare-relevant price index. This price index allows us to state the household's problem in terms of aggregate final goods.

The main decision for the households is the allocation of wealth between consumption and the savings in different investment vehicles. These decisions are guided by the following Euler equations:

$$D_t^h \;:\; \frac{\lambda_t}{R_t} = \beta \sum_{s_{t+1}|s_t} \frac{\Pr(s_{t+1}|s_t)\, V_1\left(W_{t+1}, z_{t+1}, \gamma_{t+1}, \eta_{t+1}\right)}{P_{t+1}} + \gamma_t v'\left(D_t^h/P_t\right)\frac{1}{P_t},$$

$$\Lambda_t \;:\; \frac{\lambda_t}{R_t^\Lambda} = \beta \sum_{s_{t+1}|s_t} \frac{\Pr(s_{t+1}|s_t)\, V_1\left(W_{t+1}, z_{t+1}, \gamma_{t+1}, \eta_{t+1}\right)}{P_{t+1}}.$$

The first equation compares the costs and benefits of demand deposits. The cost is the marginal utility of foregone consumption today. The future benefit is the nominal return R_t accruing in all future states and the marginal utility from deposits. The second equation relates the marginal utility cost of buying a long-term bond today to its expected future nominal return.

From these conditions we can deduce that the spread between the corporate bonds and deposits

[8] Based on our calibration, government-insured deposits constitute a sizable part of bond funding in our model. Thus the assumption that the government insures a part of the funds that are not run-prone seems reasonable.

[9] It is important to note that bank owners are assumed to be liable for bond repayments because they might be forced to inject equity funds into banks in the case of a bank run.

on the supply side is determined by the liquidity premium:

$$\left(\frac{R_t^A}{R_t} - 1\right) = \frac{R_t^A \gamma_t}{\lambda_t} v'\left(D_t^h / P_t\right).$$

Preferences are assumed to have the following functional form:

$$u\left(C_t\right) + \gamma_t v\left(D_t^h / P_t\right) = \frac{\left(C_t\right)^{1-\sigma} - 1}{1-\sigma} + \gamma_t \frac{\left(D_t^h / P_t\right)^{1-\sigma} - 1}{1-\sigma}$$

Notice that both consumption demand and liquidity demand have the same intertemporal elasticity of substitution $\frac{1}{\sigma}$. We imposed this restriction on the utility function to allow for a balanced growth path in the presence of trend productivity growth.[10]

2.2 Financial sector and production of intermediate goods

There is a financial sector that consists of an equal number of banks and patient investors, a measure one of each. The banks are funded by demand deposits D_t^B and nominal bonds A_t^B, while the patient investors are funded by bonds A_t^P only. The banks immediately invest all of the raised funds $\left(\frac{D_t^B}{P_t R_t} + \frac{A_t^B}{P_t R_t^A}\right)$ into illiquid capital of intermediate-good producers, funded by the banks. In case of a bank run, the banks must liquidate some of their assets to pay off their demand deposit liabilities. When that happens, the banks sell their assets to patient investors, who buy assets from distressed banks at a fire-sale discount. The remaining funds of patient investors are then loaned out to the intermediate-good producers funded by the patient investors.

Appendix 8.1 and Figure 4 detail the timing of events in this economy. Here, we give a brief outline focusing on the resolution of uncertainty regarding bank runs.

There are three events that can occur, after banks lend all their funds to intermediate-good producers, $K_t^B = \frac{1}{P_t}\left(\frac{D_t^B}{R_t} + \frac{A_t^B}{R_t^A}\right)$. Namely:

1. With probability $\phi_1 = (1-p)$, there are no problems and $z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta}$ of intermediate-good output is produced at the beginning of period $t+1$.

2. With probability p, a publicly observed distress signal is received, which leads households to withdraw their demand deposits. Since banks have already invested their available funds, they need to sell some or all of their assets to pay D_t^B to the holders of demand deposits, i.e. the household. Patient investors are able to buy those assets because they still have liquid funds in hand, given that their investments take place after the distress signal is received.

   (a) With probability $\phi_2 = p(1-q)$, the intermediate firms funded by the banking sector do not experience any problems and production runs smoothly. The intermediate good

[10] Our estimate of the persistence parameter of this process suggested a unit root.

output of the bank-funded sector is $z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta}$, with some part of the output now owned by patient investors.

(b) Disaster strikes with probability $\phi_3 = pq$, in which case a share $(1-\varphi)$ of output is lost. The banking sector produces $\varphi z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta}$ of intermediate goods. In addition, a fraction $(1-\varphi)$ of the undepreciated capital $(1-\delta)K_t^B$ is also lost.

We can summarize the intermediate-good production funded by banks as follows:

$$F^B\left(K_t^B, L_{t+1}^B, z_{t+1}|s\right) = \left\{ \begin{array}{cc} z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta} & \phi_1 = (1-p) \\ z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta} \quad \text{with probability} & \phi_2 = p(1-q) \\ \varphi z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta} & \phi_3 = pq \end{array} \right\}.$$

The patient investor's production function itself is unaffected by the realization of a shock specific to the banking sector. However, in the case of a bank run, patient investors buy assets from distressed banks. Thus the output of the patient investors' projects is

$$F^P\left(K_t^P, L_{t+1}^P, z_{t+1}|s\right) = \left\{ \begin{array}{cc} z_{t+1}\left(K_t^P\right)^{\theta}\left(L_{t+1}^P\right)^{1-\theta} & \phi_1 = (1-p) \\ \quad \text{with probability} & \\ z_{t+1}\left(K_t^P - \frac{D_t^P}{P_t}\right)^{\theta}\left(L_{t+1}^P\right)^{1-\theta} & \phi_2 + \phi_3 = p \end{array} \right\},$$

where $\frac{D_t^P}{P_t}$ is the amount of real funds transferred from patient investors to banks in exchange for claims to their output. It is noteworthy that a bank run that does not end in disaster still entails an economic loss because investment resources are withdrawn from production reducing the productive capacity of the economy. More specifically, the households simply store withdrawn deposits until next period $(t+1)$, realizing a real return of 1 per unit.[11]

Given that technologies funded by banks and by patient investors produce the same intermediate good, they face the same relative price of intermediate goods denoted by Q_{t+1} and are exposed to the same aggregate productivity process:

$$\log\left(z_{t+1}\right) = \log\left(z_t\right) + \varepsilon_{z,t+1},\ \varepsilon_{z,t+1} \tilde{} N\left(0, \sigma_z^2\right).$$

2.2.1 Banking sector's problem

As in Stein (2012), we assume that banks are funded through bonds and demand deposits. Demand deposits are risk-free and can be withdrawn at any time. Should households withdraw their demand deposits prematurely, the resulting fire sale leaves banks with insufficient funds to pay bond-holders.

[11] Alternatively, we could assume that demand deposits withdrawn during a fire-sale event are subject to depreciation.

In this case, either bank equity holders have to inject additional funds or the government has to step in.

Since demand deposits are risk-free, banks face a collateral constraint on the quantity of demand deposits they can issue, which ensures that the banks can pay their demand deposits given the fire-sale price of their assets. This collateral constraint gives rise to a standard pecuniary externality because the banks treat the fire-sale price as independent of their own actions.

In case of a bank run, the maximum amount of funds a bank can obtain by selling claims to all of its future revenues in a non-disaster state (s_2) is limited by:

$$\kappa_t E_t \left(\zeta_{t+1} \left((1-\delta) K_t^B + \theta Q_{t+1} z_{t+1} \left(K_t^B \right)^\theta \right) | s_2 \right)$$

where κ_t is the fire-sale discount price. There are two processes that matter directly, the idiosyncratic (bank-specific) revenue shock $\zeta_{t+1} \tilde{} \log N\left(0, (\sigma_\zeta)^2\right)$ realizes at the same time as the aggregate productivity shock $\varepsilon_{z,t+1}$ and captures the cross-sectional variation in the revenue of banks. Notice that the labour share of output $(1-\theta) Q_{t+1} z_{t+1} \left(K_t^B \right)^\theta$ has been removed from the expected revenue.

The fire-sale price κ_t will be determined in equilibrium and depend on the choices made by banks and patient investors. Specifically, a very high share of short-term funding of banks will make fire sales very costly from a bank's perspective. Similarly, a very high amount of patient investor capital will drive up the fire-sale price.

Given the risk short-term funding represents, the question arises as to why banks would want to use it? The main advantage of short-term funding is that it is cheaper than bond funding and does not create any major problems in normal times. Since bank runs are low probability events, the risk seems to be very small, and the pecuniary externality leads banks to overuse them from society's perspective.

By limiting claims only to a non-disaster state, we implicitly assume that banks can only reliably pledge their revenue if a disaster is averted. In case of a disaster, the government steps in, making patient investor's claims to banks' assets junior to the public's claims. This possibility reduces the value of bank assets to the patient investors.[12]

To summarize, the maximum amount a bank can raise in the case of a bank run is:

$$\kappa_t E_t \left(\zeta_{t+1} \left((1-\delta) K_t^B + Q_{t+1} \theta z_{t+1} \left(K_t^B \right)^\theta \right) | s_2 \right) \equiv D_t^{\max} \geq \frac{D_t^B}{P_t}.$$

Based on this, the representative banker's problem can be expressed in terms of expected next

[12] Alternatively, we could allow banks to pledge the value of their assets even in disaster situations. However, based on our calibration results this would imply a very high collateral value and lead to too much short-term borrowing as a share of total bank funding compared with what the data illustrate.

period payoff as follows:[13]

$$\max_{\substack{K_t^B,\, D_t^B \\ A_t^B}} E_t \left(\begin{array}{l} \phi_1 \left[\lambda_{t+1} \left(\begin{array}{c} P_{t+1}\varphi_1\zeta_{t+1}\left[(1-\delta)K_t^B + Q_{t+1}z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta}\right] \\ -A_t^B - D_t^B - P_{t+1}w_{t+1}^B L_{t+1}^B \end{array} \right) |s_1 \right] \\ +\phi_2 \left[\lambda_{t+1} \left(\begin{array}{c} P_{t+1}\varphi_2\zeta_{t+1}\left[(1-\delta)K_t^B + Q_{t+1}z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta}\right] \\ -A_t^B - D_t^B + \left(1-\frac{1}{\kappa_t}\right)D_t^B - P_{t+1}w_{t+1}^B L_{t+1}^B \end{array} \right) |s_2 \right] \\ +\phi_3 \left[\lambda_{t+1} \left(\begin{array}{c} P_{t+1}\varphi_3\zeta_{t+1}\left[(1-\delta)K_t^B + Q_{t+1}z_{t+1}\left(K_t^B\right)^{\theta}\left(L_{t+1}^B\right)^{1-\theta}\right] \\ -A_t^B - D_t^B + D_t^B - P_{t+1}w_{t+1}^B L_{t+1}^B \end{array} \right) |s_3 \right] \end{array} \right)$$

$$s.t.$$

$$K_t^B \leq \frac{1}{P_t}\left(\frac{A_t^B}{R_t^A} + \frac{D_t^B}{R_t}\right)$$

$$\frac{D_t^B}{P_t} \leq D_t^{\max},$$

where λ_{t+1} is the representative household's state-contingent marginal utility value of additional nominal income, Q_{t+1} is a state-contingent price of intermediate goods relative to consumption goods, and w_{t+1}^B is a state-contingent real wage paid by bank-funded intermediate-goods producers to the representative household. Banks also face the borrowing constraints, $D_t^B/P_t \leq D_t^{\max}$, on their short-term liabilities.

The main FOC's can be written as relationships between the interest rate spread and the short-term funding constraint:

$$\frac{R_t^A}{R_t} - 1 \geq \frac{E_t\left(\phi_2\left[\lambda_{t+1}\left(1-\frac{1}{\kappa_t}\right)|s_2\right] + \phi_3\left[\lambda_{t+1}|s_3\right]\right)}{E_t\left(\sum_{i=1}^{3}\phi_i\left[\lambda_{t+1}|s_i\right]\right)} \tag{5}$$

$$E_t\left(\begin{array}{c} \sum_{i=1}^{2}\phi_i\left[\lambda_{t+1}\pi_{t+1}\varphi_i\left[(1-\delta) + Q_{t+1}\theta z_{t+1}\left(K_t^B\right)^{\theta-1}\right]|s_i\right] \\ +\phi_3\left[\lambda_{t+1}\pi_{t+1}\varphi_3\left[(1-\delta) + Q_{t+1}\theta z_{t+1}\left(K_t^B\right)^{\theta-1}\right]|s_2\right] \end{array} \right) \leq E_{z_{t+1}|z_t}\left(\sum_{i=1}^{3}\phi_i\left[\lambda_{t+1}|s_i\right]\right) R_t^A \tag{6}$$

From equation 5, we know that the borrowing constraint only binds if there is a sufficiently large interest rate spread or, in other words, if the household demand for short-term funding is strong enough.

From equation 6, we know that the presence of a sufficiently sizable spread affects the scale of bank-funded production.

[13] For details regarding the banks' problem, see section 8.2 of the Appendix.

2.2.2 Patient investor's problem

The patient investors raise $K_t^P = \frac{A_t^P}{R_t^A P_t}$ units of resources in the bond market. When the public signal regarding the bank run state becomes apparent, the patient investors still have liquid funds in hand, which can be used to buy distressed assets from the banking sector.[14] Patient investors buy claims from all banks, thus diversifying away the idiosyncratic risk. Independently of the public signal, patient investors can always invest in their project. The revenue from the projects is given by $Q_{t+1}F^P\left(K_t^P, L_{t+1}^P, z_{t+1}\right)$.

The problem of a patient investor is:

$$\max_{\substack{D_t^P, K_t^P \\ A_t^P}} E_t \left(\begin{array}{l} \phi_1 \left[\lambda_{t+1} \left(P_{t+1} \left[(1-\delta) K_t^P + Q_{t+1} z_{t+1} \left(K_t^P\right)^{\theta} \left(L_{t+1}^P\right)^{1-\theta} \right] - A_t^P - P_{t+1} w_{t+1}^P L_{t+1}^P \right) | s_1 \right] \\ +\phi_2 \left[\lambda_{t+1} \left(\begin{array}{c} P_{t+1} \left[(1-\delta) \left(K_t^P - \frac{D_t^P}{P_t} \right) + Q_{t+1} z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta} \left(L_{t+1}^P\right)^{1-\theta} \right] \\ -A_t^P + \frac{D_t^P}{\kappa_t} - P_{t+1} w_{t+1}^P L_{t+1}^P \end{array} \right) | s_2 \right] \\ +\phi_3 \left[\lambda_{t+1} \left(\begin{array}{c} P_{t+1} \left[(1-\delta) \left(K_t^P - \frac{D_t^P}{P_t} \right) + Q_{t+1} z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta} \left(L_{t+1}^P\right)^{1-\theta} \right] \\ -A_t^P - P_{t+1} w_{t+1}^P L_{t+1}^P \end{array} \right) | s_3 \right] \end{array} \right).$$

Patient investors buy all the offered asset claims in the case of a bank run, $D_t^P = D_t^B$, thus they determine the equilibrium fire-sale price, κ_t. The fire-sale price in turn determines the expected return of these state-contingent claims $\frac{\phi_2}{\kappa_t}$.

2.2.3 Retailer's problem

$$\max_{\substack{P_{t+j}(i) \\ Y_{t+j}(i) \\ X_{t+j}(i)}} E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \left[\left(\frac{P_{t+j}(i)}{P_{t+j}} Y_{t+j}(i) - \frac{\bar{\varepsilon}-1}{\bar{\varepsilon}} Q_{t+j} X_{t+j}(i) \right) - \frac{\phi}{2} \left(\frac{P_{t+j}(i)}{\pi P_{t+j-1}(i)} - 1 \right)^2 Y_{t+j} \right]$$

subject to

$$\begin{aligned} Y_{t+j}(i) &= \left(\frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\varepsilon_{t+j}} Y_{t+j} \\ Y_{t+j}(i) &= X_{t+j}(i). \end{aligned}$$

In the retailer's problem, the constant $\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}}$ can be thought of as a production subsidy from the government, which is assumed to align the steady-state output in normal times with its efficient level.

We focus on a symmetric equilibrium and, following much of the sticky-price literature, we add

[14] Patient investors in our model purposefully hold back funds in search of more profitable investment opportunities. This is a reasonable response to the existence of uninsured risk.

a markup shock $\varepsilon_t = \bar{\varepsilon}\exp(\eta_t)$ to the optimal pricing equation. Thus we obtain:

$$\begin{aligned} 0 &= \lambda_t\left[(\bar{\varepsilon}\exp(\eta_t)-1)Y_t\left(\frac{\bar{\varepsilon}\exp(\eta_t)}{(\bar{\varepsilon}\exp(\eta_t)-1)}\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}}Q_t-1\right)-\phi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}Y_t\right] \\ &\quad +E_t\beta\lambda_{t+1}\left[\phi\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}Y_{t+1}\right], \end{aligned}$$

where

$$\eta_t = \rho_e\eta_{t-1}+\zeta_t.$$

The markup shock will be essential for matching pricing moments in the data. It also contributes to the tradeoff between price stability and financial stability because monetary policy might be confronted with high inflation, weak aggregate demand, and low short-term funding relative to total bank funding. Besides the tension between inflation and aggregate demand, there is now the additional issue of the desirable bank balance sheet composition, suggesting different potential actions regarding the interest rate.

2.3 Government

The government in this economy sets the interest rate on demand deposits according to the following rule:

$$\ln\left(\frac{R_t^D}{R_t^n}\right) = \phi_\pi\ln\left(\frac{\pi_t}{\bar{\pi}}\right)+\phi_C\ln\left(\frac{C_t}{\bar{C}}\right)+\phi_D\left(\frac{D_t^B}{\bar{D}^B}\right).$$

The rule responds to deviations of the inflation rate from its target $\bar{\pi}$ and the level of consumption relative to its steady-state value in normal times. In addition, monetary policy may react to the level of short-term funding, D_t^B, relative to its steady-state value in normal times. This interest rate R_t is specified relative to its flexible price benchmark R_t^n. We could instead define the interest rate relative to its steady state value $\bar{R}$, but in our model with large disaster shocks, the steady state is not invariant to policies. As a result, a flexible-price interest rate provides a better reference for interest rate policies.

We chose consumption as our measure of economic activity mainly for computational reasons because using output instead of consumption would require an expansion of the state space. We verify that consumption is strongly correlated with output in equilibrium.

The government also provides a subsidy to retailers of final goods to ensure an efficient steady-state output in normal times, financed by lump-sum taxes. Finally, the government sets a minimum equity constraint that captures the spirit of the Basel II style capital adequacy ratio, demanding that equity needs to be at least 8 per cent of capital at risk. In our model, losses of up to 8 per cent of risky assets (in our case, all assets) are ignored when we are computing the risk of bank defaults. This constraint doesn't directly affect banks' behaviour, but it affects the probability of a bank run, as we will explain in section 2.5. This is a short-cut approach to capture the stabilizing effect of bank regulation.

2.4 Market clearing

There are seven markets in this economy: final goods market, intermediate goods market, two segmented labour markets, the short-term funding market, the long-term funding market, and the fire-sale market. All of these markets must clear.

$$\begin{aligned} C_t + K_t^B + K_t^P &= Y_t\left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2\right) + (1-\delta)\left(K_{t-1}^B + K_{t-1}^P\right), \\ &\text{if there is no bank run in } t-1. \\ C_t + K_t^B + K_t^P &= Y_t\left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2\right) + (1-\delta)\left(K_{t-1}^B + K_{t-1}^P\right) + \frac{D_t^h}{P_t} \\ &\text{if there is a bank run in } t-1, \text{ but no disaster in period } t. \\ C_t + K_t^B + K_t^P &= Y_t\left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2\right) + (1-\delta)\left(\varphi K_{t-1}^B + K_{t-1}^P\right) + \frac{D_t^h}{P_t} \\ &\text{if there is a bank run in } t-1, \text{ and a disaster in period } t. \end{aligned}$$

$$\begin{gathered} Y_t(i) = X_t^P(i) + X_t^B(i) \\ L_{t+1}^B = 1, \quad L_{t+1}^P = 2 \\ D_t^h = D_t^B \\ \left(K_t^B - \frac{D_t^B}{P_t R_t}\right) + K_t^P = \frac{A_t}{R_t^A P_t} \\ D_t^B = D_t^P \end{gathered}$$

Note that labour is distributed equally across the two sectors in equilibrium: $L_t^B = L_t^P = 1$.

2.5 Probability of a distress signal

In our model, the probability of a distress signal is a function of the state of the economy. Because we focus on fundamental economic risk, we assume that the probability of a distress signal depends on the probability that an individual bank becomes insolvent. Since bank-specific productivity shocks are independently and identically distributed across banks, all banks face the same default probability in a given period. Specifically, we have:

$$\begin{aligned} \Pr\left(\text{insolvency}_{t+1}\right) = (1-p)\Pr(\Pi_{t+1}^B(s_{t+1}) \leq -\upsilon K_t^B | s_1) \\ +p(1-q)\Pr(\Pi_{t+1}^B(s_{t+1}) \leq -\upsilon K_t^B | s_2) \\ +pq\Pr(\Pi_{t+1}^B(s_{t+1}) \leq -\upsilon K_t^B | s_3), \end{aligned} \tag{7}$$

where Π^B_{t+1} is an individual bank's profit in period $t+1$ defined as follows

$$\Pi^B_{t+1}(s_{t+1}) = \left\{ \begin{array}{ll} \zeta_{t+1}\left[(1-\delta)K^B_t + Q_{t+1}z_{t+1}\left(K^B_t\right)^{\theta}\left(L^B_{t+1}\right)^{1-\theta}\right] - w^B_{t+1}L^B_{t+1} & \\ \quad -\frac{A^B_t}{P_{t+1}} - \frac{D^B_t}{P_{t+1}} & |s_{t+1}=1 \\ \zeta_{t+1}\left[(1-\delta)K^B_t + Q_{t+1}z_{t+1}\left(K^B_t\right)^{\theta}\left(L^B_{t+1}\right)^{1-\theta}\right] - w^B_{t+1}L^B_{t+1} & \\ \quad -\frac{A^B_t}{P_{t+1}} - \frac{D^B_t}{P_{t+1}}\frac{1}{\kappa_t} & |s_{t+1}=2 \\ \varphi\zeta_{t+1}\left[(1-\delta)K^B_t + Q_{t+1}z_{t+1}\left(K^B_t\right)^{\theta}\left(L^B_{t+1}\right)^{1-\theta}\right] - w^B_{t+1}L^B_{t+1} & \\ \quad -\frac{A^B_t}{P_{t+1}} - \frac{D^B_t}{P_{t+1}}\frac{1}{\kappa_t} & |s_{t+1}=3 \end{array} \right\}. \quad (8)$$

The variable $\zeta_{t+1} \tilde{} \log N\left(0,(\sigma_\zeta)^2\right)$ is an idiosyncratic shock to a bank's revenue. All banks are assumed to face this bank-specific revenue risk. This shock realizes at the same time as the aggregate productivity shock z_{t+1} and captures the cross-sectional variation in the revenue of banks.

The value $0 < \kappa_t \leq 1$ is the fire-sale discount that entices patient investors to buy distressed assets from the banks.

In the equation 7, we take the presence of the regulatory requirement into account by ignoring losses of up to $-vK^B_t$ where $v = 0.08$.

To link the insolvency probability to the probability of a distress signal, we simply assume that $p = \Pr\left(\text{insolvency}_{t+1}\right)$. Under this assumption, equation 7 can be transformed into a more insightful expression:

$$p = \frac{\Pr(\Pi^B(s_1) \leq -vK^B_t)}{\Pr(\Pi^B(s_1) \leq -vK^B_t) + (1-q)\Pr\left(\Pi^B(s_2) \leq -vK^B_t\right) + q\Pr\left(\Pi^B_{t+1}(s_3) \leq -vK^B_t\right)}.$$

Hence, our association between distress and insolvency has clear implications for the possibility of bank runs. For bank runs to occur, it is essential that a positive measure of insolvencies be possible in good times, i.e. $\Pr\left(\Pi^B(s_1) \leq -vK^B_t\right) > 0$. In our economy, it is thus important that the 8 per cent regulatory equity requirement be insufficient to absorb all shocks. To achieve this, we need to allow for idiosyncratic risk ζ_{t+1} to affect the gross return of banks.[15] In the calibration section, we use the balance sheet data on the variation in profit rates per unit of assets to assess the amount of idiosyncratic revenue risk faced by US banks.

It is important to mention that the effect of monetary policy on the distress probability is ex ante ambiguous. In good times, lower deposit rates lead to higher profits since banks save on interest payments. However, these lower rates may also result in banks having a stronger demand for short-term funding and hence to costlier fire sales in the case of a bank run. We find that the impact of lower rates on the distress probability depends on the endogenous response of the interest rate spread R^A_t/R_t, which can either increase or decrease depending on the changes in the supply of short-term funding. In our experiments, the total impact of policy rates on the average distress

[15] A similar insight led Angeloni and Faia (2013) to augment the standard productivity shock with a sizable idiosyncratic component.

probability was found to be fairly small.[16] As a result, monetary policy will be less concerned with influencing the average default probability, focusing instead on containing the expected costs of bank runs and on supporting investment before and after crises occur.

2.6 Monetary policy, bank-run risk and short-term funding

In our model, the banking disaster can only happen after a bank run. With the endogenous probability of bank runs, monetary policy can reduce the probability of fire sales and banking disasters by making bank runs less likely. A reduction in the probability of bank runs could be achieved by restricting the amount of short-term liabilities issued by banks. While our assumption that banking disasters cannot happen without bank runs is strong, it gives monetary policy the best chance to achieve its financial stability objectives by discouraging short-term funding and ensuing fire sales. However, since the representative household derives utility from demand deposits, a complete elimination of short-term funding can not be optimal. The central bank in our model must strike a balance between the utility benefits from demand deposits and their potential costs in a bank run situation. The price stability concerns and the maintenance of a functional savings-investment channel add additional constraints on the ability of the central bank to attain financial stability.

The policy rate R_t in our model is the only tool for achieving the central bank objectives. However, it is not the policy rate per se but the interest rate spread $\left(\frac{R_t^A}{R_t} - 1\right)$ that affects the willingness of banks and of the representative household to create demand deposit liabilities. When the central bank changes the short-term rate R_t, it affects the spread between the bond rate, R_t^A, and the policy rate, R_t. A narrower spread has two opposing effects on the quantity of demand deposits. On the one hand, banks have fewer incentives to offer demand deposits because the cost advantage of demand deposits has decreased in favour of bonds. On the other hand, the representative household has an incentive to increase its demand deposit holdings since the opportunity cost has diminished as a result of the tighter spread. This effect on household deposits follows from the household's optimality condition:

$$\left(\frac{R_t^A}{R_t} - 1\right) = \frac{R_t^A \gamma_t}{\lambda_t} \left(D_t^h / P_t\right)^{-\sigma}.$$

If the spread term on the left-hand side decreases, and the ratio $\frac{R_t^A \gamma_t}{\lambda_t}$ does not change much, the $\left(D_t^h/P_t\right)^{-\sigma}$ –term on the right-hand side must decline, which can only happen if the real deposits of the household, D_t^h/P_t, increases.

In our experiments, the negative impact of tighter spreads on the banks' incentive to offer demand deposits is dominated by the stimulating effect on the supply of household demand deposits.

[16] The empirical crisis literature, see for example Schularick and Taylor (2012) and Bauer (2014), also finds a fairly low baseline probability similar to ours. Moreover, using such empirical estimates, Svensson (2016) suggests that the benefits to monetary policy leaning are small, in part because of the weak response of the probability function.

2.7 Solving for the equilibrium

As the economy confronts very large shocks, we solve the model non-linearly using the endogenous grid points method first proposed by Carroll (2006). Specifically, we define a grid over the state of the economy, S, which includes real wealth, W, the liquidity demand shock, γ, and the cost-push shock, η. Then we use the solution to the social planner problem, see section 8.4 of the Appendix, as an initial guess for the consumption policy function $C(S)$ as well as for the functions $p(S)$, $\pi(S)$ and $Q(S)$. Starting with this initial guess, we iterate backward by solving the portfolio problems and generating updates for the endogenous grid over W.

An important aspect of the paper is the optimization of policy rule coefficients. Here we always start our optimization problems with a global search algorithm (simulated annealing) to avoid local maxima. After the algorithm has sufficiently converged, we turn to a local optimization routine, the Nelder-Mead algorithm, using the candidate solution from the simulated annealing.

3 Calibration

We calibrate the model to match key characteristics of the US economy and its banking sector. The calibration section proceeds in two steps. First, we explain the choice of some basic parameters, then we explain the estimation of the remaining ones. In all of our simulations, we take an inflation target of 2 per cent per year as given.[17]

3.1 Basic parameters

The basic parameters are $(\theta, \delta, \sigma, \bar{\varepsilon}, \phi)$ and their values as well as the rationale for setting them are summarized in Table 1.

For the capital income share parameter θ, we use the average capital income share based on the National Income and Product Accounts (NIPA) data as provided by the Bureau of Economic Analysis (BEA). After splitting entrepreneurial income into its labour and capital components, we reduce aggregate income by indirect business taxes and add subsidies to obtain a more accurate output measure. We find for the period 1952–2013: $\theta = 0.364$.

The depreciation rate of capital is chosen so as to minimize the distance between the BEA real total private asset stock and one constructed based on BEA quarterly real business investment from 1952–2013. We obtain $\delta = 0.025$.

Next, we chose the intertemporal elasticity of substitution at $\frac{1}{\sigma} = 0.5$, as suggested in the literature, see the cross-country study by Havrnek et al. (2013), for example.

Next, we have to determine two price setting parameters. Specifically, we set the CES substitution coefficient that determines the retailers' markup to $\bar{\varepsilon} = 8$, implying an average markup of

[17] Doing a welfare comparison across alternative targets, we found that the optimal target in the model is between 2 and 2.5 per cent, though the welfare differences are small.

14.29 per cent. Given this choice, we then determine the Rotemberg adjustment cost parameter $\phi = 25$. This value is consistent with the estimates of the slope of the Phillips curve available in the literature and gives the values of the real cost of inflation that are reasonable relative to Lucas's estimate of the cost of business cycles.

3.2 Estimation

We estimate the remaining 11 parameters using a minimum distance estimation procedure, which aims to match simulated model moments to their counterparts in the data. To do so, we solve:

$$\min_{v} g(\xi)' g(\xi)$$

where we have $\xi = \left(\beta, q, \phi_\pi, \sigma_z, \varphi, \bar{\gamma}, \rho_\gamma, \sigma_\gamma, \rho_\eta, \sigma_\eta\right)'$ and $g(\xi) = \frac{m(\xi)}{m} - 1$, with m being the data moments and $\hat{m}(\xi)$ the model moments based on a 100,000 period simulation. Unless otherwise stated, the data moments are based on US time series for the period from 1986Q1 to 2007Q3. The comparable model moments are obtained from simulations without financial crisis situations. For both model and data moments, we analyze Hodrick-Prescott filtered log_e time series. The moments for model and data are listed in Table 3, the parameter estimates in Table 2.

We will now list the 11 target moments and the associated parameter estimates. The stated association is loose but helps to think about the relevance of the moment choice. Our moments reflect the key aspects of the paper, namely the role of short-term funding in the economy and its relationship with the business cycle. Thus we have both interest rate and short-term funding as well as output-related moments.

Interest rate moments

- Average real corporate bond yields for bonds with AAA rating by Moody's, 3.78 per cent annualized → household discount rate $\beta = 0.9899$.[18]
- Average spread between the bond and the fed funds rate adjusted for the term premium, 1.53 per cent annualized → household demand for liquidity parameter $\bar{\gamma} = 0.02524$.
- First order autocorrelation of interest rate spreads, $\frac{R^B}{R^D}$, 0.63 → persistence parameter of liquidity demand shock process, $\rho_\gamma = 0.9971$.
- First order autocorrelation of policy rate, 0.54 → the policy rule coefficient on inflation, $\phi_\pi = 1.715$.

Short-term funding moments

[18] We unfortunately only had AAA-rated bond yields for financial intermediaries for a short period. During that period, however, they behaved similarly to those of non-financial AAA-rated corporate bonds.

To obtain a short-term funding series, we use the flow of funds data for deposit-taking institutions as well as for brokers/dealers. Both groups of financial institutions were heavy users of short-term wholesale funding. To measure the short-term funds exposed to bank runs, we combine wholesale short-term funding (e.g. repos) with uninsured retail deposits.

- Average share of short-term funding of private deposit taking institutions and brokers and dealers, 35.34 per cent → standard deviation of idiosyncratic productivity shocks, $\sigma_\varsigma = 2.9$ per cent.

- Standard deviation of short-term funding, 2.04 per cent → the standard deviation of innovations to interest rate spread shocks, $\sigma_\gamma = 0.0009$.

Output moments

- Standard deviation of real output, 1.33 per cent → standard deviation of innovations to total factor productivity (TFP), $\sigma_z = 0.0109$.

- Drop in real output during the Great Depression, 0.26 → The fraction of banks' revenue remaining, if a disaster state realizes, $\varphi = 0.7086$.

Correlation of a financial variable with output

- Correlation of output and interest rate spreads, −0.57. It should be noted that, based on simulations, this moment is the most informative with respect to the business cycle variation in the distress probability.

Price stability moments

- Standard deviation of the consumption less energy and food price inflation in the NIPA, 0.26 per cent → Standard deviation of the markup shock innovations, $\sigma_\eta = 0.2027$.

- First order autocorrelation of the consumption less energy and food price inflation in the NIPA, 0.79 → Persistence parameter of the markup shock process, $\rho_\eta = 0.8383$.

Overall, the calibration procedure resulted in a reasonable fit of data and model moments. It is worthwhile to discuss some moments and parameter estimates in greater detail.

In the model, the correlation of output and the interest rate spread is of particular relevance for capturing the business cycle variation in the probability of a bank run. The data moment of −0.57 is fairly close to the model moment of −0.53. We also closely match the share of short-term funding used by banks in the model (data: 35.3 per cent, model: 34.6 per cent). This moment is important because it partially determines the importance of the pecuniary externality in the model together with the spread moments, which we also matched reasonably well.

Furthermore, we determine the standard deviation of the idiosyncratic shock to banks' revenue as $\sigma_\zeta = 0.029$ per cent. This implies an average probability of a distress signal of $\bar{p} = 0.55$ per cent, or an average time between crises of 45.5 years. To get independent evidence regarding the idiosyncratic shock, we looked at the cross-sectional balance sheet data for publicly traded banks in the United States. The data we collected contained the profit rate per unit of assets because this measure accords well with the net rate of investment return in the model. The top panel of Figure 5 shows the cross-sectional standard deviation of profits per unit of asset from 2000 onward. We find that our estimate for idiosyncratic risk is clearly in the range of possible values. Furthermore, we find that the share of firms facing losses in excess of the regulatory equity holdings reaches a value of up to 0.66 per cent during the Great Recession, highlighting a sizable amount of risk similar to that in our model.

The model understates the standard deviation of short-term funding at 1.47 per cent, while the data value is 2.04 per cent.

The current calibration struggles the most with the inflation moments. Especially the standard deviation of inflation in the model at 0.12 per cent is half the actual standard deviation. Likely this is related to a fairly high value of $\phi_\pi = 1.7$ compared with Taylor's estimate of 1.5.[19]

There are four additional moments that we wish to report. First, the high first-order autocorrelation of real output pushed us to assume a random walk for the productivity process $\ln(z_t)$. To accommodate the unit root of productivity, we renormalized all variables with respect to productivity, which was possible as the model admits a balanced growth path.

Second, we initially also estimated the Taylor coefficient on the consumption gap, ϕ_C. However, the estimation converged to a value very close to zero, leading us to set the parameter to zero and re-estimate. The moment we tried to match with ϕ_C was the correlation of output and short-term funding, which is 0.71 in the data. The moment implied by the model is 0.9.

A further moment that we did not target is the drop in real output during the Great Recession. The related model moment is an output drop during a financial crisis without disaster. In the model it is marginally higher than that found in the data, 7.65 per cent compared with 6.26 per cent.

Finally, the standard deviation of the policy rate in the model is 0.22 per cent, compared with 0.33 per cent in the data.

4 Results

Before we go into detail, it is instructive to consider the main findings. In the model, monetary policy alone doesn't solve the bank-run problem. This is consistent with the fairly broad policymaker consensus that financial regulation is the first line of defence against a crisis. However, even taking this limit as given, monetary policy can influence the conditions under which risk increases and reduce fluctuations in risk, which it optimally does in our economy.

[19] We could not push the value of ϕ_π lower owing to convergence issues.

Depending on the variables reflected in the policy rule, different strategies become available. With a traditional inflation-output reaction function, monetary policy focuses more on overall economic stabilization. When short-term funding is added to the policy reaction function, the optimized policy leads to a substantial welfare improvement. The systematic reaction to short-term funding achieves a better stabilization of the economy, specifically during periods of financial crisis. This is achieved by a reduction in the volume of short-term funding relative to patient investors' capital. This reduction lowers the average output loss by 1 per cent during a crisis. Moreover, it leads to consistently higher savings.

To understand the importance of monetary policy, we consider three economies, each associated with a particular version of the monetary policy rule

$$\ln\left(\frac{R_t}{R_t^n}\right) = \phi_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_C \ln\left(\frac{C_t}{\bar{C}}\right) + \phi_D \left(\frac{D_t}{\bar{D}}\right). \tag{9}$$

We analyze three special cases:

- $(\phi_\pi, \phi_C, \phi_D) = (\phi_\pi^{BM}, 0, 0)$: The benchmark monetary policy (BMP) from the calibration.
- $(\phi_\pi, \phi_C, \phi_D) = (\phi_\pi^{ROMP}, \phi_C^{ROMP}, 0)$: The restricted (welfare) optimized monetary policy (ROMP), which is prevented from reacting directly to short-term funding.
- $(\phi_\pi, \phi_C, \phi_D) = (\phi_\pi^{OMP}, \phi_C^{OMP}, \phi_D^{OMP})$: The welfare optimized unrestricted monetary policy (OMP), allowing a reaction to all three variables.

Coefficients in the OMP rule feature a much more aggressive response to inflation fluctuations compared with the BMP, see Table 4. The OMP also responds to fluctuations in banks' short-term funding positions as well as to fluctuations of aggregate consumption and improves welfare by 0.43 per cent in terms of Lifetime-Consumption Equivalents (LTCE), illustrated by the welfare loss numbers in columns 2 and 3 of Table 4.[20]

The restricted central bank responds even more strongly to fluctuations in inflation, while accommodating fluctuations in consumption, see Table 4. However, the restriction is welfare costly with a loss relative to the unrestricted rule of 0.22 per cent of LTCE.

One other important distinction between the ROMP and the OMP rules is that the average policy rate is lower for the unrestricted case, see Table 4. The interest rate difference is, on average, 43 basis points, but it is much more pronounced during recession, when the OMP implies an interest rate difference of 1.23 per cent that is lower than the ROMP. The other interest rates also reflect this difference in policy rates. The bond rate is 51 basis points lower, on average, under the OMP, with a more pronounced difference of 1.35 per cent during recessions. Thus we conclude that the ability to react to the short-term credit allows the central bank to pursue a more accommodative interest rate policy, especially during recoveries from recessions. This policy accommodation allows

[20] See Appendix 8.5 for details regarding our welfare measure.

for higher savings rates and higher wealth. The wealth is 4.3 per cent higher on average under the OMP, see Table 4, with an even larger difference of more than 6 per cent during recessions. The average savings rate is 1 percentage point higher under the OMP compared with the ROMP.

One insight we gained from these experiments, is that it is hard to grasp the impact of monetary policy on the economy by looking just at the first and second moments of variables. Our model features strong non-linearities resulting from tail events associated with bank runs, fire sales and output losses. The non-linear global solution method that we employ allows us to assess the effect of changes in monetary policy not only in the vicinity of the stochastic steady state, but also during the low probability events associated with bank runs. However, since the model economy spends most of its time around its stochastic steady state, the impact of policy variation on first and second moments is relatively minor. For this reason, we find it more instructive to look at the distributions of the relevant variables. This approach demostrates more clearly how OMP and ROMP manage risks and economic disruptions during tail events.

4.1 Comparing the distributions under different policies

In this section we compare different policies focusing on the overall distributions of endogenous variables, like short-term funding and inflation.

4.1.1 The optimized policy compared with the benchmark policy

When comparing the implications of the OMP with the BMP, i.e. the calibrated rule, we learn that the OMP contains the average crisis probability in the economy much more effectively, generating higher wealth and comparable average inflation. It does so by containing banks' short-term funding and allowing inflation to deviate strongly on the downside at times, creating a downside-skewed inflation distribution.

Going into details, Figures 6 and 7 show the distributions of inflation and short-term funding relative to patient investors' assets plotted against the variation in the crisis probability. The crisis probability is always on the vertical axis. Blue crosses and lines represent simulation results and distributions from the BMP economy. Red circles and lines represent simulation results and distributions from the economy with the OMP. Looking at the distributions of the crisis probability plotted to the right of scatterplots, we can immediately see that the OMP successfully tightens the crisis probability distribution and lowers the highest observed crisis probability: the mode of the probability distribution shifts down by approximately 1 basis point, and the distribution becomes much more concentrated. It is also noteworthy that the simulations generate a range of crisis probabilities between 53.8 and 60 basis points for OMP compared with 54.2–64 basis points for BMP. This might seem like a small difference, but in terms of the expected time between crisis events, the probability differences are fairly sizable at 42–47 years compared with 39–46 years. Thus the OMP can delay the expected arrival time of a crisis by 1 to 3 years compared with the BMP.

The probability density functions on the horizontal axis of Figure 6 show that the distribution of inflation also becomes much more concentrated under the OMP. At the same time, the flat downward slopping shape of the red scatterplot on Figure 6 highlights that the optimizing central bank might confront massive declines in inflation,[21] at the same time as the crisis probability starts rising. Notice that this is consistent with Issing's (2003) idea that central banks might be willing to deviate from their price stability objective during times of financial stress. The key to the welfare success of the OMP is the higher average wealth and a stronger concentration of wealth around the stochastic steady state, see Table 4.

How does monetary policy achieve the higher wealth level? Essentially, an interest rate response to short-term funding leads to an adjustment in private sector expectations, which improves the central bank's tradeoff between the level of interest rates and the amount of short-term liabilities issued by banks. This favourable adjustment in expectations is similar to that realized by an aggressive response to inflation, which improves the tradeoff between inflation and output. The mere threat of a strong reaction to short-term funding is sufficient to moderate the accumulation of short-term liabilities. In our optimized policy simulations, the ability of the central bank to control short-term funding lowers the average interest rates relative to the benchmark economy. The lower borrowing rates stimulate investment and wealth accumulation, leading to a substantial welfare gain. In addition, the OMP lowers the risk of a crisis by restricting the short-term funding share of banks and thus creating a safer environment for investment. Figure 7 plots the distribution of banks' short-term funding normalized by the total assets of patient investors. The clear separation between the blue and the red clouds suggests that one way in which the optimized policy manages to reduce the crisis probability and improve welfare is by reducing the size of short-term liabilities relative to the capacity of patient investors to provide liquidity in the case of bank runs. This highlights that it is useful from a financial stability standpoint to measure short-term funding relative to the available outside liquidity because this indicator provides a good assessment of potential fire-sale conditions. What the OMP does very well is ensure that this ratio does not get very big and that it actually contracts with the economy in times of crisis. This is very much in contrast to the BMP, which features the highest share of short-term funding relative to outside liquidity at the same time as the crisis probability is near its peak.

Returning to Table 4, we can see that policy rates under the OMP and the ROMP are lower on average than under the BMP, and do not vary as much. However, the standard deviations of inflation and output are higher under the two optimized policies. This confirms that first and second moments are not necessarily the best statistics to consider when aiming for welfare maximization in a world with extreme events.

[21] A scatterplot for expected inflation looks nearly identical to the one for realized inflation.

4.1.2 The restricted optimized compared with the unrestricted optimized rule

A comparison of the ROMP and the OMP reveals that the ROMP is able to achieve a lower standard deviation of inflation, output and the policy rate at the same time it reduces the average probability of a crisis. However, it does so at the expense of a lower average level of wealth and, as a result, lower welfare. The key obstacle the ROMP can not overcome is that without direct policy response to short-term funding, it must accept higher levels of short-term funding, costlier bank runs, a riskier investment climate and lower overall savings in the economy.

As in the previous section, the Figures 8 and 9 show the distributions of inflation and short-term funding relative to the assets owned by patient investors, plotted against the variation in the crisis probability. The crisis probability is always on the vertical axis. Blue crosses and lines represent simulation results and distributions for the economy with under the ROMP. Red circles and lines repeat simulation results and distributions from the economy with the OMP.

Looking at the distributions of inflation and the crisis probability in Figure 8, we can see that the ROMP is more conservative than the OMP. Specifically, ROMP admits fewer and less sizable declines of the inflation rate, while at the same time slightly reduces the crisis probability. A comparison of columns 3 and 4 from Table 4 confirms that the average crisis probability is slightly lower under ROMP, and the volatilities of inflation, output and the policy rate are also smaller under ROMP (see the numbers in the bottom three rows). Yet, the welfare loss is 22 basis points greater. This is due to higher steady state wealth the OMP is able to generate as it successfully reduces the amount of short-term funding relative to the funds available to patient investors, as shown in Figure 9.

Overall this experiment suggests that, in our economy, the ability of the central bank to change interest rates in response to fluctuations in banks' short-term liabilities is important for welfare outcomes. Without this flexibility, the central bank becomes more cautious and less accommodative with respect to risk taking and wealth accumulation. However, in contrast to the concept of "leaning against the wind," the OMP coefficient on short-term funding is actually negative (see column 3 of Table 4). This means the policy rate is decreasing in short-term funding. As we discussed in the introduction, this policy response to short-term funding has a stabilizing effect on the amount of short-term liabilities because the household supply of short-term funding is more responsive to changes in the interest rate spreads than the banks' demand. In addition the adjustment in private expectations regarding policy aggressiveness toward short-term debt allows the unrestricted optimized policy to lower the policy rates and, at the same time, tighten the interest-rate spread. Columns 3 and 4 in Table 4 confirm that the average spreads are lower under the OMP because the higher investment rate has a moderating effect on long-term interest rates.

It is important to note, that the negative coefficient on consumption in the restricted optimal policy, $\phi_C^{ROMP} < 0$, emulates the effects of the short-term funding coefficient under OMP, $\phi_D^{OMP} <$

0.[22] Both coefficients help moderate the sharp falls in interest rates in response to the very large declines in inflation after bank-run events. Specifically, both consumption and demand deposits fall below their steady-state values in normal times after such crisis periods. Thus the regular Taylor coefficients would lead to a massive decline of the policy rate, which would lead to large spreads, stimulate short-term funding and increase the risk of another bank run. The negative coefficients in the rules reduce the effect, trading a strong deviation of inflation and output against the lower risk of a crisis. The impact of the coefficients on the policy rate can be clearly seen in the decompositions of the policy rule effects in Figure 10. Note that the coefficient values for ROMP and OMP rules are as shown in columns 3 and 4 of Table 4.

4.2 Does monetary policy matter for risk?

None of the monetary policies we consider changes much the average probability of a crisis. This, however, does not necessarily mean that the monetary policy has no influence on risk. To get a better understanding of the way the OMP responds and deals with risk, we focus attention on periods with a high probability of a crisis, i.e. high-risk periods. As a reference, we use the ROMP, which generates the lowest mean distress probability. For the ROMP, we determine the 99.5th percentile of its distress probability distribution, which is the value of distress probability $\bar{p}$, such that only 0.5 per cent of simulation periods under ROMP have a distress probability above that level. Second, we sample the distributions of distress probabilities under the BMP and under the OMP and report the fraction of periods their distress probabilities exceed the same level $\bar{p}$. This is our measure of the prevalence of high-risk periods under different monetary policies. Under the OMP, 2 per cent of the periods fall in that high-risk group, and under the BMP, the economy spends a massive 14.8 per cent of the time in the high-risk zone. Thus clearly both the OMP and the ROMP reduce the risk of a crisis relative to the BMP.

Finally, for all three policies we examine first and second moments of inflation, output, interest rates and other variables to see how high-risk periods differ across policies. Table 5 compares some of the summary statistics for periods of high risk and all other periods.

Are high-risk periods special in a systematic way?

We find the following main characteristics of the high-risk periods. First, the economy is below steady state in terms of wealth, output and inflation. Second, the cost-push shocks are far away from their means, though the direction of the gap is policy-specific. Third, the economic outlook regarding output and inflation, which are expected to remain substantially below target, is weak. Fourth, all policy rates are stimulating during those periods, with the OMP creating the most aggressive stimulus. Finally, the short-term funding is low under all policies with the OMP inducing

[22] The strongest impact of the negative coefficients is present in the aftermath of a bank-run event. A worthwhile extension of the analysis would be to develop wealth-dependent policy coefficients. This would likely lead to more standard responses to economic conditions when wealth is close to the steady state in normal times and provide clear prescriptions during disasters episodes.

the strongest deviation from the steady state in normal times.

Do policies influence high-risk periods?

It is clear that the OMP manages tail risk in the sense that it restricts the amount of short-term funding, thus avoiding costly downturns, and reduces the probability mass in the upper tail of the crisis risk distribution. For example, both OMP and ROMP induce lower short-term funding compared with the BMP, in both absolute and relative terms. The lower amounts of short-term funding lead to a tighter constraint on short-term funding, as evidenced by the higher values of Lagrange multipliers on the constraint. Also, the optimized policies admit high risk of distress during periods with severe cost-push shocks, while managing to avoid high-risk during other periods.

5 Conclusions

We extend and carefully calibrate the Stein (2012) model in which banks' funding choices can lead to bank runs and costly fire sales. In this setup, we assess the impact of variations in monetary policy rules on the probability of bank runs and on the severity of output losses induced by such runs. We find that it is welfare-beneficial for central banks to respond aggressively to inflation, while at the same time responding with a negative coefficient to the variation in banks' short-term liabilities. The OMP leads to a welfare gain equivalent to nearly 0.22 per cent of life-time consumption, relative to the ROMP, which does not admit a direct response to short-term funding. Thus our model provides additional support to interest rate rules that are sensitive to some measure of financial activity. Another finding based on our model is that the ROMP is slightly more aggressive on inflation fluctuations than the OMP. It attains lower volatilities of inflation, output and policy rates while reducing the average probability of financial crises at the same time. We also find that the considered optimized monetary policy rules, OMP and ROMP, in our environment use part of their influence on the reduction of tail risk by trying to keep the crisis probability distribution close to its mean. This is done even at the expense of occasionally compromising price stability.

One of the premises of our analysis is that regulatory tools are insufficient to control the risks arising from banks' funding risks. However, these tools are the natural first line of defence against such risks. As pointed out in Stein (2013), appropriately set reserve requirements bear much promise in this regard. Contemplating the interaction between regulatory tools (e.g. leverage regulation) and monetary policy, when handling residual risks in our setup, would likely be a fruitful extension of the presented analysis.

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6 Tables

Table 1: Basic model parameters

Parameter	Value	Determination
Capital income share	$\theta = 0.364$	BEA NIPA, average 1952–2013
Depreciation rate	$\delta = 0.025$	BEA NIPA, 1952–2013
Intertemporal elasticity of substitution	$\sigma = 2$	Fixed
Elasticity of substitution intermediate goods	$\bar{\varepsilon} = 8$	Match average markup in the United States
Rotemberg inflation adjustment cost	$\phi = 25$	Estimates of the Phillips curve slope

Table 2: Estimated model parameters, for period from 1986Q1 to 2007Q3

Concept	Parameter	
	General	value
		Model
Household discount factor	β	0.9924
Idiosyncratic return risk of banks	σ_ζ	0.0290
Probability of disaster given a distress signal	q	0.6155
Policy rule weight on inflation	ϕ_π	1.7150
Std. of productivity innovations	σ_z	0.0109
Output drop during disaster distress state	φ	0.7086
Household demand for liquidity	$\bar{\gamma}$	0.0252
Persistence of liquidity demand shocks	ρ_γ	0.9971
Std. of liquidity demand shock innovations	σ_γ	0.0009
Cost push shock persistence	ρ_η	0.2028
Std. of cost push shock innovations	σ_η	0.8383

Table 3: MOMENTS IN MODEL AND DATA FOR THE PERIOD FROM 1986Q1–2007Q3

TARGET MOMENTS	DATA	MODEL
Targets		
Average interest rate spread, y/y annualized	1.52%	1.28%
Average real bond return, y/y annualized	3.78%	4.23%
Autocorrelation of spread	0.63	0.77
Autocorrelation of policy rate	0.54	0.64
Average share of short-term funding in total bank funding	35.34%	34.57%
Standard deviation short-term funding	2.04%	1.47%
Standard deviation of output	1.33%	1.33%
Drop of real output during Great Depression	0.26	0.28
Correlation output, interest rate spread	-0.57	-0.53
Standard deviation of consumer price inflation	0.26%	0.12%
Autocorrelation of consumer price inflation	0.79	0.64
Other moments		
Drop of GDP in 2008–09	6.26%	7.64%
Standard deviation of policy rate	0.33%	0.22%
Correlation output, short-term funding	0.71	0.90
Autocorrelation of output	0.93	0.84

Table 4: POLICY REGIMES IN THE ENDOGENOUS PROBABILITY MODEL

1	2	3	4
MOMENT	BENCHMARK	OPTIMIZED MON. POLICY	RESTRICTED OPTIMIZED
Welfare loss, LTCE	0.744	0.310	0.530
Mon. policy coefficient on			
inflation	1.751	3.857	3.996
Short-term funding (STF)	0	-0.191	0
consumption	0	0.067	-0.199
Average:			
crisis probab.	0.558	0.546	0.543
inflation	1.902	0.626	1.444
policy rate	3.295	2.540	2.973
spread	1.292	1.195	1.274
fire-sale price	0.343	0.338	0.341
savings rate	0.288	0.300	0.290
STF/Total Bank Funding	0.345	0.340	0.343
Normalized levels of:			
Lagrange multiplier	100	97.65	93.96
output	100	102.1	100.5
wealth	100	105.6	101.2
Standard deviation of			
crisis probabability	0.008407	0.008406	0.005780
inflation	0.812	1.100	0.583
output	1.472	1.526	1.455
policy rate	1.427	0.974	0.616

Note: Welfare is in per cent of LTCE relative to social planner.

Average inflation, policy rate and spread are annualized, in per cent.

All standard deviations are in per cent. Standard deviations of inflation, output and policy rate are annualized.

Table 5: AVERAGE ECONOMIC CONDITIONS AND POLICY ACTIONS BY CRISIS PROBABILITY, HIGH-RISK PERIODS, ALL OTHER PERIODS. HERE A HIGH-RISK PERIOD IS DEFINED BY THE 0.995TH PERCENTILE OF THE DISTRESS PROBABILITY DISTRIBUTION OF THE ROMP

1	2		3		4	
	BENCHMARK		OPTIMIZED MON.POLICY		RESTRICTED OPTIMIZED	
	High risk	Other days	High risk	Other days	High risk	Other days
TFP shocks	1.00	1.00	1.00	1.00	1.00	1.00
Cost-push shocks	0.69	1.07	1.32	1.00	1.35	1.00
Wealth	1.87	2.01	1.72	2.10	1.62	2.01
Inflation	1.63	1.96	-3.27	1.48	-1.35	1.65
Policy rate	2.82	3.40	-0.07	2.57	2.24	2.98
Output	0.161	0.166	0.155	0.169	0.150	0.167
Spread	1.40	1.27	1.47	1.19	1.61	1.27
Lagr.multiplier	0.25	0.21	0.32	0.21	0.35	0.22
Expected inflation	1.75	1.93	-3.20	1.44	-1.28	1.63
Expected output	0.164	0.168	0.158	0.171	0.155	0.168
STF/PI capital	0.343	0.342	0.333	0.336	0.340	0.341
Pr(Crisis), in %	0.573	0.555	0.569	0.545	0.569	0.543

Note: Average inflation, policy rate and spread are annualized, in per cent.

7 Figures

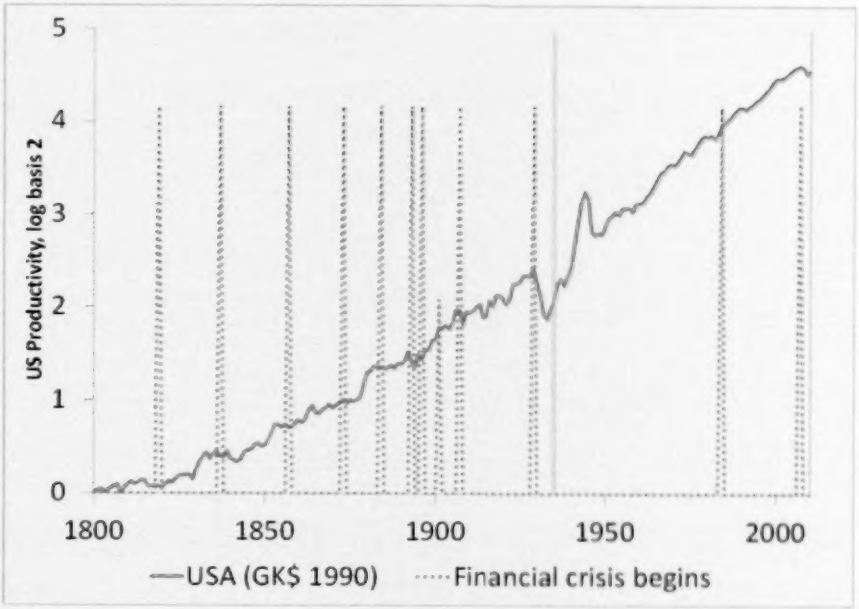

Figure 1: Financial crises events (dashed spikes show begining period) and real activity from 1800 to 2010. Source: Angus Maddison Database and Reinhart and Rogoff (2011).

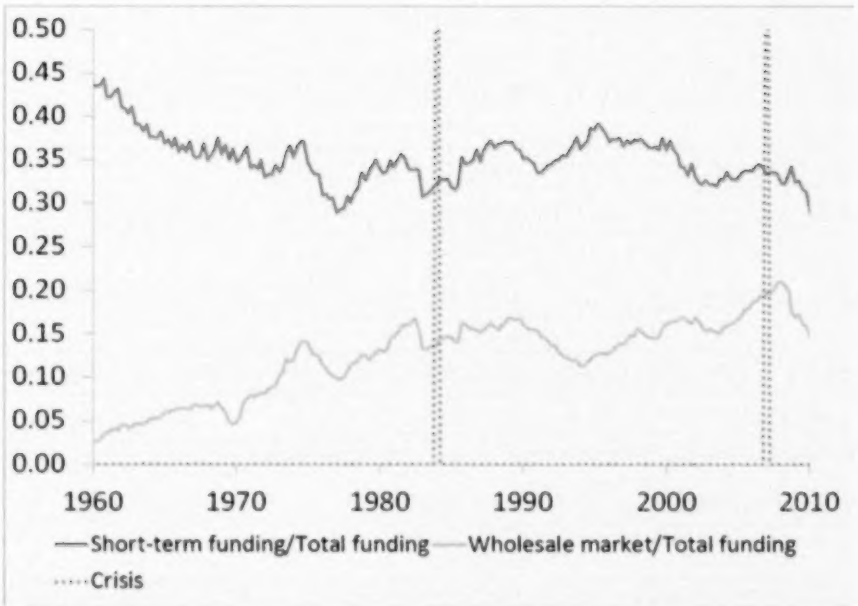

Figure 2: Short-term funding by deposit taking institutions and brokers and dealers in the US from 1960 to 2010. Source: Flow of Funds and Reinhart and Rogoff (2011).

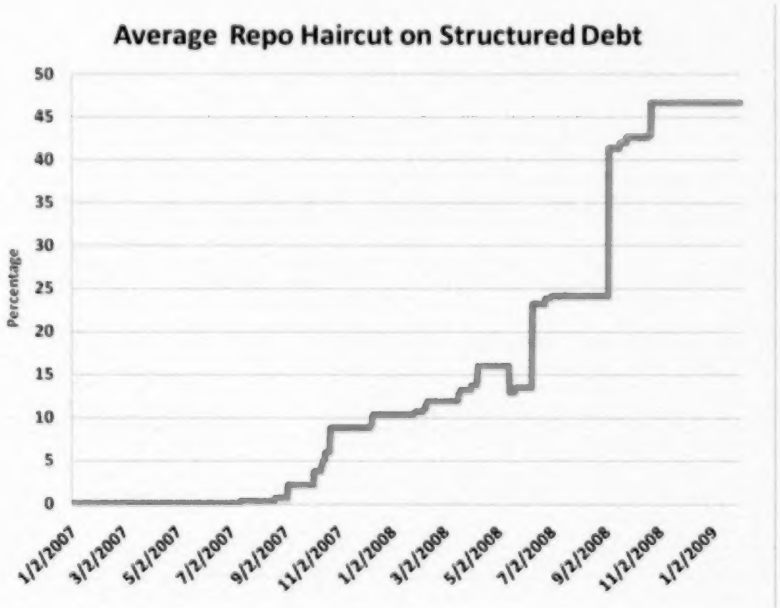

Figure 3: Average haircut of structured repos at one repo desk during the 2007–09 financial crisis. Source: Gorton and Metrick (2012).

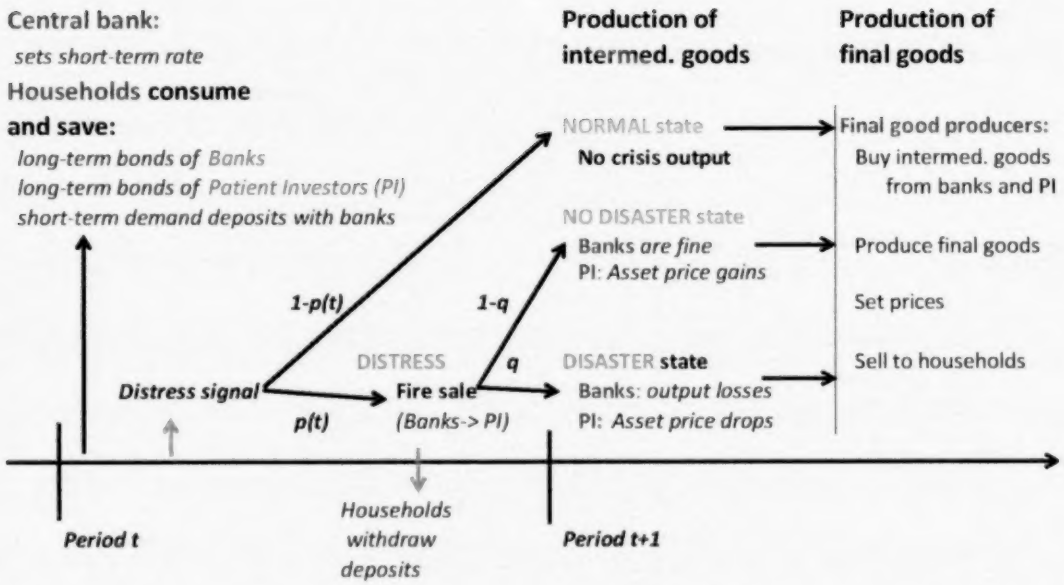

Figure 4: The timing of events and resolution of uncertainty in the model economy

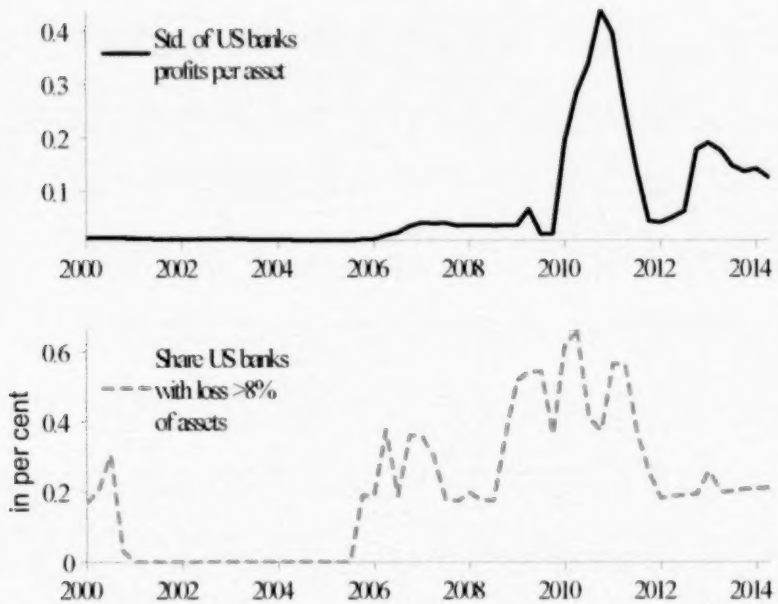

Figure 5: Cross-sectional data for publicly traded US financial institutions. Source: Bloomberg.

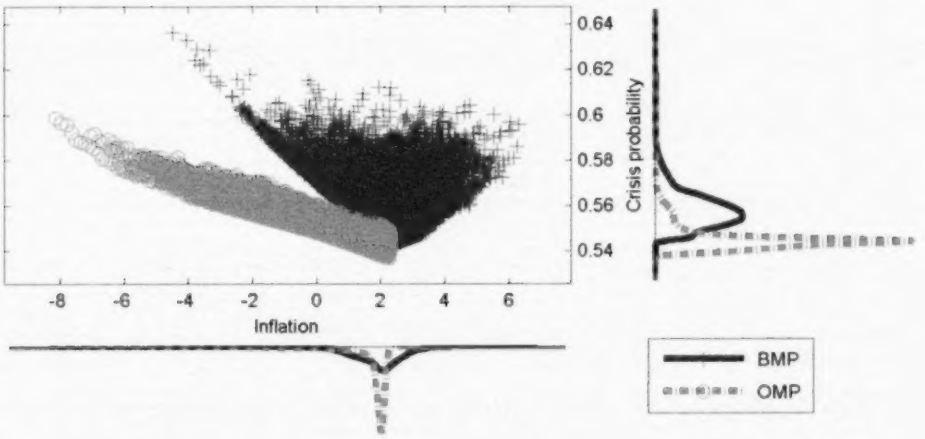

Figure 6: Simulation results for the benchmark monetary policy (BMP) and the optimized monetary policy (OMP). Net annualized inflation rate on the horizontal scale and the financial crisis probability on the vertical. Both variables are in per cent.

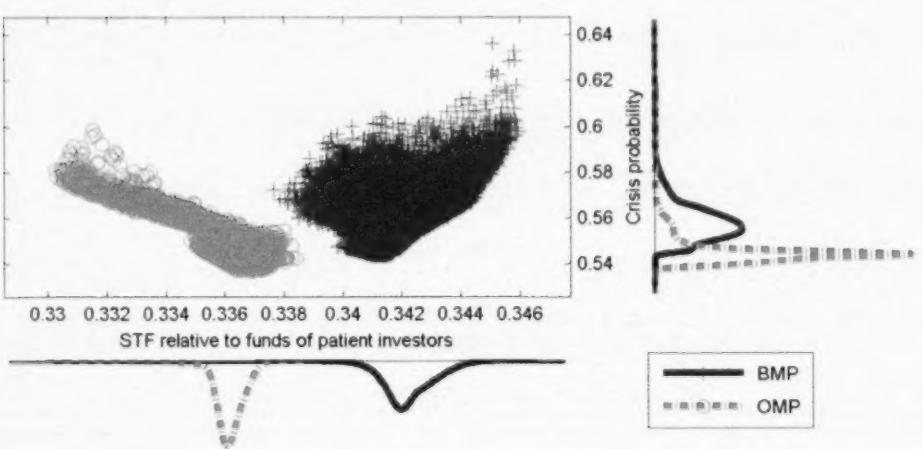

Figure 7: Simulation results for the benchmark monetary policy (BMP) and the optimized monetary policy (OMP). Short-term funding (STF) relative to assets of patient investor on the horizontal scale and the financial crisis probability on the vertical. The probability is in per cent.

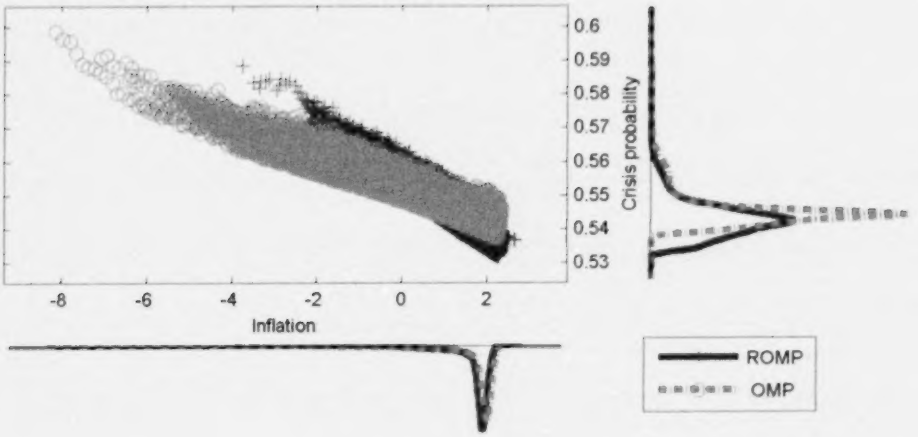

Figure 8: Simulation results for the restricted optimized monetary policy (ROMP) and the unrestricted one (OMP). Net annualized inflation rate on the horizontal scale and the financial crisis probability on the vertical. Both variables are in per cent.

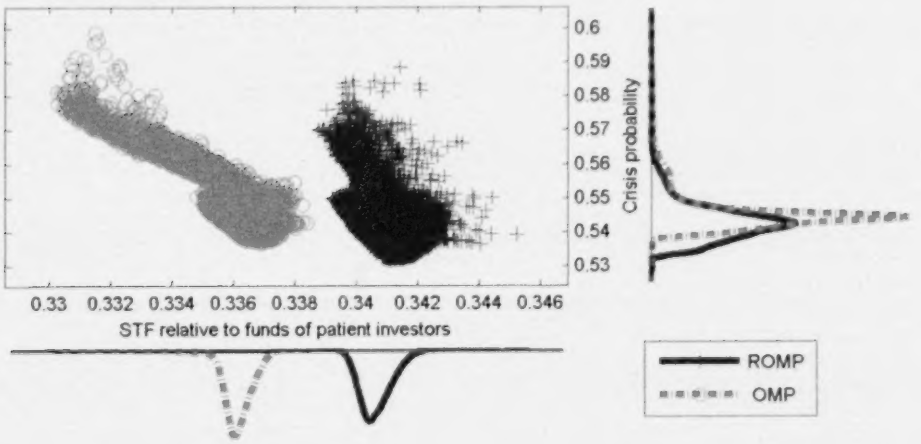

Figure 9: Simulation results for the restricted optimized monetary policy (ROMP) and the unrestricted one (OMP). Short-term funding (STF) relative to assets of patient investors on the horizontal scale and the financial crisis probability on the vertical. The probability is in per cent.

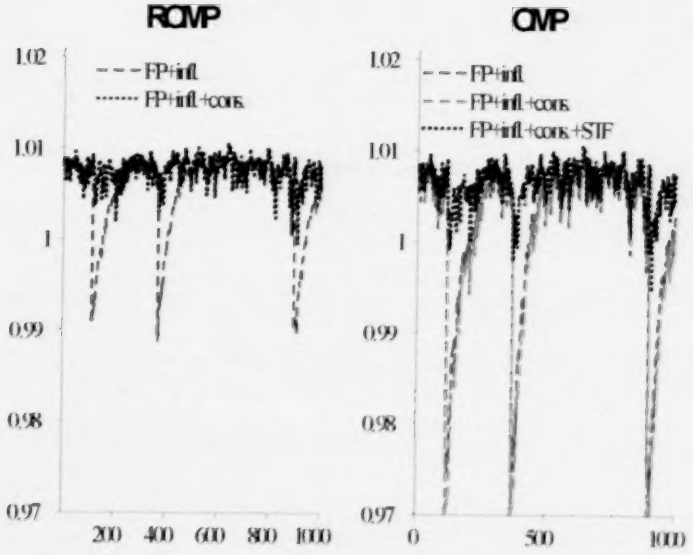

Figure 10: The decomposition of the effects of the individual terms in the policy rule. $\ln\left(\frac{R_t}{R_t^n}\right) = \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_C \ln\left(\frac{C_t}{C}\right) + \phi_D \left(\frac{D_t}{D}\right)$ on the policy rate R_t. FP stands for the natural (flexible price) interest rate benchmark, R_t^n. FP + infl. stands for the interest rate profile implied by the following rule: $\ln\left(\frac{R_t}{R_t^n}\right) = \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right)$. FP + infl. + cons. adds the consumption term to the policy rule: $\ln\left(\frac{R_t}{R_t^n}\right) = \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_C \ln\left(\frac{C_t}{C}\right)$. Finally the FP + infl. + cons. + STF curve on the right panel shows the policy rate implied by the full OMP policy rule, $\ln\left(\frac{R_t}{R_t^n}\right) = \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_C \ln\left(\frac{C_t}{C}\right) + \phi_D \left(\frac{D_t}{D}\right)$. Note that the coefficient values for ROMP and OMP rules are as shown in columns 3 and 4 of Table 4.

8 Appendix

8.1 The timing of events

The period t starts with the realization of shocks. Afterward, production takes place and the returns on investment are paid. We will give more details about the settlements of assets and liabilities further below. For now let us focus on the events that happen after the period t production takes place and all claims are settled. Figure 4 shows the timing of the events in the model covering the time frame between the period t consumption and investment decisions, and the period $t+1$ production and liability repayment outcomes. Let us state the timing of these events step-by-step:

1. The central bank sets the current short-term interest rate target R_t, and the representative household divides its wealth W_t between consumption C_t and investments: D_t^h, A_t^h. The household immediately derives utility $\gamma_t v\left(\frac{D_t^h}{P_t}\right)$ from its demand deposits.

2. The banks invest all of their funds into illiquid capital of their intermediate-good firms, $K_t^B = \frac{1}{P_t}\left(\frac{D_t^B}{R_t} + \frac{A_t^B}{R_t^B}\right)$. The patient investors wait.

3. A publicly observable signal arrives with probability $p_t(=p(t))$. It signals that the banking system can be in distress. With probability $(1-p_t)$ the banking system is sound.

4. If the distress signal is positive, households run on banks to withdraw demand deposits D_t^h. The banks sell claims to the output of their intermediate-good firms at the fire-sale discount price κ_t. The patient investors buy distressed assets by giving up $\frac{D_t^P}{P_t}$ of the investment funds they have available. The rest of the funds $\left(A_t^P - \frac{D_t^P}{P_t}\right)$ is rented as capital to intermediate-good firms owned by patient investors. If the distress signal is negative, there is no bank run and no fire sales of assets.

5. In case of a bank run, the households end the period t with D_t^h held in storage; A_t^B of bank bonds; and A_t^P of patient investors' bonds. If there is no bank run in period t, the banks still hold demand deposits on their balance sheet at the end of period t.

6. Period $t+1$ starts with the realization of four shocks: (i) the productivity of intermediate-good firms z_{t+1}, (ii) the idiosyncratic revenue shocks of banks ζ_{t+1}, (iii) the mark-up shock, η_{t+1}, (iv) the liquidity preference shock of the households γ_{t+1}.

7. The production of intermediate goods takes place, and it becomes clear whether the current state is a disaster, happening with probability q, or not. Note that the disaster in period $t+1$ is only possible, if there was a bank run in period t. If a disaster happens, the fraction φ of banks' output and undepreciated capital becomes useless, and fire-sale assets owned by patient investors are worthless to them. In any case, the fraction $(1-\theta)$ of output is paid to households as labour income.

8. The monopolistically competitive retailers buy intermediate goods at the relative price Q_{t+1} and produce differentiated final goods, while taking in account the household demand for their products, as well as the government production subsidy, which reduces their marginal cost by the factor $\frac{\varepsilon-1}{\varepsilon}$. The final-good firms set their optimal prices.

9. All claims and liabilities are settled. The patient investors and banks pay bond returns to the households. Should the funds be insufficient to cover them, equity holders and the government might step in, depending on the size of the revenue shortfall. All banks are dissolved regardless of their bankruptcy status and a new set of banks is created. The representative household's wealth W_{t+1} is determined by the payoffs on all of its assets, as well as by the lump-sum transfers from the government, as shown in the budget constraint, equation (2).

8.2 Bank problem

Equivalently, we can state the problem as:

$$\max_{K_t^B,\, d_t} P_t E_t \left(\begin{array}{c} \phi_1 \left[\lambda_{t+1} \left(\begin{array}{c} \pi_{t+1}\varphi_1\zeta_{t+1} \left[(1-\delta) K_t^B + Q_{t+1} z_{t+1} \left(K_t^B\right)^{\theta} \left(L_{t+1}^B\right)^{1-\theta} \right] \\ -R_t^A K_t^B + \left(R_t^A - R_t\right) K_t^B d_t - \pi_{t+1} w_{t+1}^B L_{t+1}^B \end{array} \right) | s_1 \right] \\ +\phi_2 \left[\lambda_{t+1} \left(\begin{array}{c} \pi_{t+1}\varphi_2\zeta_{t+1} \left[(1-\delta) K_t^B + Q_{t+1} z_{t+1} \left(K_t^B\right)^{\theta} \left(L_{t+1}^B\right)^{1-\theta} \right] - R_t^A K_t^B \\ + \left(R_t^A - R_t\right) K_t^B d_t + \left(1 - \frac{1}{\kappa_t}\right) R_t K_t^B d_t - \pi_{t+1} w_{t+1}^B L_{t+1}^B \end{array} \right) | s_2 \right] \\ +\phi_3 \left[\lambda_{t+1} \left(\begin{array}{c} \pi_{t+1}\varphi_3\zeta_{t+1} \left[(1-\delta) K_t^B + Q_{t+1} z_{t+1} \left(K_t^B\right)^{\theta} \left(L_{t+1}^B\right)^{1-\theta} \right] - R_t^A K_t^B \\ + \left(R_t^A - R_t\right) K_t^B d_t + R_t K_t^B d_t - \pi_{t+1} w_{t+1}^B L_{t+1}^B \end{array} \right) | s_3 \right] \end{array} \right) \tag{10}$$

$$s.t.$$

$$d_t \leq d_t^{\max},$$

where we have:

Variable	Parameter	Explanation
K_t^B		Total investment in bank projects
$d_t = \frac{D_t^B/R_t}{P_t K_t^B}$		Share of short-term funding
R_t^A		Return to long-term borrowing
R_t		Return to demand deposits
w_t^B		Wage per labour unit
λ_{t+1}		Household's marginal utility of income
Q_{t+1}		The relative price of intermediate goods
z_{t+1}		Aggregate productivity
ζ_{t+1}		Idiosyncractic shocks to gross revenue
	$d^{\max}$	Maximum share of short-term financing
	p	Probability of a distress signal
	φ	Share of output received in case of disaster
	δ	Capital depreciation rate

The FOC's for the banks' problem can be stated as:

$$L_{t+1}^B : w_{t+1}^B = \varphi_i \zeta_{t+1} (1-\theta) Q_{t+1} z_{t+1} \left(K_t^B\right)^{\theta}$$

$$K_t^B : E_t \left(\begin{array}{c} \sum_{i=1}^{3} \phi_i \left[\lambda_{t+1} \left(\pi_{t+1} \varphi_i \left[(1-\delta) + Q_{t+1} \theta z_{t+1} \left(K_t^B\right)^{\theta-1} \right] - R_t^A + \left(R_t^A - R_t\right) d_t \right) | s_i \right] \\ +\phi_2 \left[\lambda_{t+1} \left(1 - \frac{1}{\kappa_t} \right) R_t d_t | s_2 \right] \\ +\phi_3 \left[\lambda_{t+1} R_t d_t | s_3 \right] \end{array} \right) = 0$$

$$d_t : E_t \left(\begin{array}{c} \sum_{i=1}^{3} \phi_i \left[\lambda_{t+1} \left(R_t^A - R_t \right) | s_i \right] \\ +\phi_2 \left[\lambda_{t+1} \left(1 - \frac{1}{\kappa_t} \right) R_t | s_2 \right] \\ +\phi_3 \left[\lambda_{t+1} R_t | s_3 \right] \end{array} \right) = \frac{\mu_t}{K_t^B}$$

$$\mu_t \geq 0, \ \mu_t \left(d_t^{\max} - d_t \right) = 0, \ d_t^{\max} \geq d_t.$$

8.3 Patient investor's problem

We can re-state the patient investor's problem in the main text as follows.

$$\max_{D_t^P, K_t^P} E_t \left(\begin{array}{c} \phi_1 \left[\lambda_{t+1} \left(P_{t+1} \left[(1-\delta) K_t^P + Q_{t+1} z_{t+1} \left(K_t^P \right)^{\theta} \left(L_{t+1}^P \right)^{1-\theta} \right] - R_t^A P_t K_t^P - P_{t+1} w_{t+1}^P L_{t+1}^P \right) | s_1 \right] \\ + \phi_2 \left[\lambda_{t+1} \left(\begin{array}{c} P_{t+1} \left[(1-\delta) \left(K_t^P - \frac{D_t^P}{P_t} \right) + Q_{t+1} z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta} \left(L_{t+1}^P \right)^{1-\theta} \right] \\ - R_t^A P_t K_t^P + \frac{D_t^P}{\kappa_t} - P_{t+1} w_{t+1}^P L_{t+1}^P \end{array} \right) | s_2 \right] \\ + \phi_3 \left[\lambda_{t+1} \left(\begin{array}{c} P_{t+1} \left[(1-\delta) \left(K_t^P - \frac{D_t^P}{P_t} \right) + Q_{t+1} z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta} \left(L_{t+1}^P \right)^{1-\theta} \right] \\ - R_t^A P_t K_t^P - P_{t+1} w_{t+1}^P L_{t+1}^P \end{array} \right) | s_3 \right] \end{array} \right).$$

From this, we obtain the first order necessary conditions:

$$D_t^P : E_t \left(\begin{array}{c} \phi_2 \left[\lambda_{t+1} \left(P_{t+1} Q_{t+1} \left[-\frac{1}{P_t} (1-\delta) - \frac{1}{P_t} \theta z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta-1} \right] + \frac{1}{\kappa_t} \right) | s_2 \right] \\ + \phi_3 \left[\lambda_{t+1} \left(P_{t+1} Q_{t+1} \left[-\frac{1}{P_t} (1-\delta) - \frac{1}{P_t} \theta z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta-1} \right] \right) | s_3 \right] + \xi_t \end{array} \right) = 0$$

$$K_t^P : E_t \left(\begin{array}{c} \phi_1 \left[\lambda_{t+1} \left(P_{t+1} \left[(1-\delta) + Q_{t+1} \theta z_{t+1} \left(K_t^P \right)^{\theta-1} \right] - R_t^A P_t \right) | s_1 \right] \\ + \phi_2 \left[\lambda_{t+1} \left(P_{t+1} \left[(1-\delta) + Q_{t+1} \theta z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta-1} \right] - R_t^A P_t \right) | s_2 \right] \\ + \phi_3 \left[\lambda_{t+1} \left(P_{t+1} \left[(1-\delta) + Q_{t+1} \theta \varphi z_{t+1} \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta-1} \right] - R_t^A P_t \right) | s_3 \right] \end{array} \right) = 0$$

$$L_{t+1}^P : w_{t+1}^P = Q_{t+1} z_{t+1} (1-\theta) \left(K_t^P - \frac{D_t^P}{P_t} \right)^{\theta}$$

where $K_t^P \geq D_t^P \geq 0$.

8.4 Social planner problem

The social planner problem without financial constraints can be expressed as follows:

$$V(W,z,\gamma)=\max u(C)+\gamma v(D)+\beta E\left[\sum_{s}\Pr(s)V(W_s',z',\gamma')\right]$$

$$s.t.$$

$$C+K^B+K^P\leq W$$

$$W_s'\leq\left\{\begin{array}{c}(1-\delta)\left(K^B\right)+z'\left(K^B\right)^{\theta}+(1-\delta)K^P+z'\left(K^P\right)^{\theta}\\(1-\delta)\left(K^B\right)+z'\left(K^B\right)^{\theta}+(1-\delta)\left(K^P-D\right)+z'\left(K^P-D\right)^{\theta}+D\\\lambda\left((1-\delta)\left(K^B\right)+z'\left(K^B\right)^{\theta}\right)+(1-\delta)\left(K^P-D\right)+z'\left(K^P-D\right)^{\theta}+D\end{array}\right\}$$

$$\zeta:D\leq K^B$$

$$z'=ze^{\varepsilon_z}\bar{z},\varepsilon_z\tilde{}N\left(0,\sigma_z^2\right)$$

$$\gamma'=\gamma^{\rho_\gamma}e^{\varepsilon_\gamma}\bar{\gamma},\varepsilon_\gamma\tilde{}N\left(0,\sigma_\gamma^2\right)$$

Where $K^i,i=B,P$ are the resources given to the technology directly without any insurance nature, while D are resources that are given to the bank technology but can be withdrawn in case of distress.

This problem implies the following simplified FONCs:

$$K^P:u'(C)=E\left[\lambda_{2,g}\left(1-\delta+z'\theta\left(K^P\right)^{\theta-1}\right)+\left(\lambda_{2,b}+\lambda_{2,h}\right)\left(1-\delta+z'\theta\left(K^P-D\right)^{\theta-1}\right)\right]$$

$$K^B:E\left[\begin{array}{c}\lambda_{2,g}\left(z'\theta\left(K^P\right)^{\theta-1}-z'\theta\left(K^B\right)^{\theta-1}\right)\\+\lambda_{2,b}\left(z'\theta\left(K^P-D\right)^{\theta-1}-z'\theta\left(K^B\right)^{\theta-1}\right)\\+\lambda_{2,h}\left((1-\delta)(1-\lambda)+z'\theta\left(K^P-D\right)^{\theta-1}-\lambda z'\theta\left(K^B\right)^{\theta-1}\right)\end{array}\right]-\zeta=$$

$$D:\gamma v'(D)-E\left[\left(\lambda_{2,b}+\lambda_{2,h}\right)\left(\left(1-\delta+z'\theta\left(K^P-D\right)^{\theta-1}\right)-1\right)\right]-\zeta=0$$

$$W_s':\lambda_{2,s}=\beta E\left[\Pr(s)\ V_1\left(W_s',z',\gamma'\right)\right]$$

$$\lambda_1:C+K^B+K^P=W$$

$$\lambda_{2,s}:W_s'\leq\left\{\begin{array}{c}(1-\delta)\left(K^B\right)+z'\left(K^B\right)^{\theta}+(1-\delta)K^P+z'\left(K^P\right)^{\theta}\\(1-\delta)\left(K^B\right)+z'\left(K^B\right)^{\theta}+(1-\delta)\left(K^P-D\right)+z'\left(K^P-D\right)^{\theta}+D\\\lambda\left((1-\delta)\left(K^B\right)+z'\left(K^B\right)^{\theta}\right)+(1-\delta)\left(K^P-D\right)+z'\left(K^P-D\right)^{\theta}+D\end{array}\right\}$$

$$\zeta\geq0,\zeta\left(K^B-D\right)=0,D\leq K^B$$

8.5 Life-time consumption equivalent (LTCE)

We use a LTCE measure to compare welfare across economies with various monetary policy rules. In this appendix we show how we derive and compute this measure. Suppose in period 0 we have the expected welfare in the CE and SP solutions as

$$U^{CE} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \bar{\gamma}\gamma_t \frac{\left(\frac{D_t}{P_t}\right)^{1-\sigma}}{1-\sigma} \right]$$

$$U^{SP} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \bar{\gamma}\gamma_t \frac{\left(\tilde{d}_t\right)^{1-\sigma}}{1-\sigma} \right]$$

Define LTCE measure as the value λ such that

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left((1+\lambda)\, c_t\right)^{1-\sigma}}{1-\sigma} + \bar{\gamma}\gamma_t \frac{\left(\frac{D_t}{P_t}\right)^{1-\sigma}}{1-\sigma} \right] = U^{SP}.$$

We can express λ as

$$\lambda = \left(\frac{U^{SP} - E_0 \sum_{t=0}^{\infty} \beta^t \left[\bar{\gamma}\gamma_t \frac{\left(\frac{D_t}{P_t}\right)^{1-\sigma}}{1-\sigma} \right]}{E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t)^{1-\sigma}}{1-\sigma} \right]} \right)^{\frac{1}{1-\sigma}} - 1. \tag{11}$$

In our welfare calculations we simulated each economy with the same sequence of shocks over $T = 100,000$ periods. Then we approximated λ using sample means of $\bar{\gamma}\gamma_t \frac{\left(\frac{D_t}{P_t}\right)^{1-\sigma}}{1-\sigma}$ and $\frac{(c_t)^{1-\sigma}}{1-\sigma}$ from this simulation. The resulting welfare measure takes the following form:

$$\hat{\lambda} = \left(\frac{\frac{1}{T} \sum_{t=1}^{T} \left[\frac{\tilde{c}_t^{1-\sigma}}{1-\sigma} + \bar{\gamma}\gamma_t \frac{\left(\tilde{d}_t\right)^{1-\sigma}}{1-\sigma} \right] - \frac{1}{T} \sum_{t=1}^{T} \left[\bar{\gamma}\gamma_t \frac{\left(\frac{D_t}{P_t}\right)^{1-\sigma}}{1-\sigma} \right]}{\frac{1}{T} \sum_{t=1}^{T} \left[\frac{(c_t)^{1-\sigma}}{1-\sigma} \right]} \right)^{\frac{1}{1-\sigma}} - 1. \tag{12}$$

We experimented with the sample size and found that 100,000 simulation periods were needed to obtain a reliably consistent welfare measure across policies.